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Feed-forward Carrier Frequency and Timing Synchronization for MSK Modulation *

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***Abstract-* The paper addresses the problem of carrier and timing synchronization for MSK modulation. Based on a second-order and a fourth-order statistical variable, an efficient data-aided algorithm is proposed to estimate the frequency offset and timing error, respectively. Numerical results show that the proposed algorithm achieves good performance in both AWGN and Rayleigh fading channels and it outperforms previous data-aided synchronization algorithms, even in fading channels.**

I. Introduction

Minimum Shift Keying (MSK), a Continuous Phase Modulation (CPM), has gained considerable attention in recent years with the rapid development of wireless communications. MSK conserves bandwidth and reduces energy at the same time. Furthermore, nonlinear amplifiers can be used in MSK, which makes transmission more power efficient and hence attractive for communication systems.

To demodulate the received signal correctly in the receiver, knowledge of carrier phase, symbol timing and frequency offset are required. Frequency offset and symbol timing error are the most often encountered problems in a radio communication system [1]. The optimal synchronization approach to timing and frequency offset estimation is described in [2][3], which is known as the maximum-likelihood (ML) or the maximum-a-posteriori (MAP) based joint timing and frequency offset estimation. However, it is not very practical due to its computation complexity. Consequently, several suboptimal approaches, which make tradeoff between synchronization performance and implementation complexity, have been proposed.

By means of pilot symbols, data-aided synchronization algorithms have been proposed to extract the carrier-frequency and timing information [1], [4]. In [1], the information is extracted by a differential operation and, in [4], the symbol timing information is extracted from the argument difference between every symbols, both can work well in high SNR cases. However, digital frequency discriminator employed in [1] requires higher oversampling ratio to obtain more accurate outputs, which results in greater computation complexity. The estimator provided in [4] can only estimate the symbol timing for a special case when the oversampling ratio is 4, and no frequency offset estimation algorithm is involved. Moreover, in low SNR cases, the synchronization performance of both estimators degrades dramatically. In this paper, a novel data-aided feedforward synchronization algorithm is proposed. The algorithm, which is based on two statistical variables, is computationally more efficient and achieves better performance when compared with the approaches in [1],[4].

The paper is organized as follows. In Section II the signal model is presented. In Section III, we define the two statistical variables and explain how frequency offset and symbol timing can be extracted. Simulation results are presented in Section IV and conclusions are drawn in Section V.

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II. Signal Model

Consider a narrowband MSK modulated signal transmitted through a Rayleigh fading channel, the simplified block diagram is shown in Fig.1.

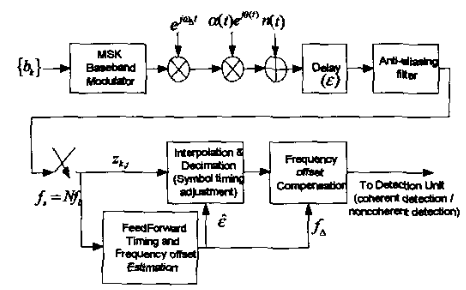

Fig.1Equivalent complex baseband signal model for a MSK system

The received MSK signal from radio frequency (RF) to baseband is oversampled, the sampled signal at time $(k+\frac{i}{N})T$ can be written as

$$z_{k,i} = \alpha_{k,i} e^{j\theta_{k,i}} e^{j\{\phi(kT+iT/N-\varepsilon T)+2\pi f_\Delta (k+i/N)T+\psi\}} + n_{k,i} \quad (1)$$

where $\{\alpha_{k,i}\}$ and $\{\theta_{k,i}\}$ are the amplitudes and phases of the fading channel, and the phase distortions are uniform distributed in $[0,2\pi)$, $\varepsilon \in (-0.5, 0.5]$ is the fraction of a symbol duration by which the received signal is time-shifted with respect to the original signal. The oversampling ratio

(OSR) is equal to N, which is equal to the ratio of the symbol period T to the sample period T_s, f_Δ is the carrier-frequency offset between the transmitter and the receiver, ψ is the initial phase offset, $\phi(t)$ is the information-bearing phase, which is defined as

$$\phi(t) = 2\pi h \sum_i b_i q(t - iT) \qquad (2)$$

where $\{b_i\}$ are independent data symbols, $q(t)$ is the phase pulse of the modulator, which for MSK is expressed as [5]

$$q(t) = \begin{cases} 0 & t < 0 \\ \frac{t}{2T} & 0 < t < T \\ \frac{1}{2} & t > T \end{cases} \qquad (3)$$

and h is the modulation index, equals to 0.5 for MSK. Thus, for MSK signal, $\phi(t)$ can be rewritten as

$$\phi(t) = \frac{\pi b_n (t - (n-1)T)}{2T} + \frac{\pi}{2} \sum_{i=0}^{n-1} b_i \qquad (4)$$

for $(n-1)T \le t \le nT$. The zero mean complex AWGN noise $n_{k,i}$ satisfies

$$\begin{aligned} &E\{\mathrm{Re}(n_{k,i}) \cdot \mathrm{Re}(n_{k',i'})\} = E\{\mathrm{Im}(n_{k,i}) \cdot \mathrm{Im}(n_{k',i'})\} \\ &= \frac{N_0}{2} \delta(k - k')\delta(i - i') \end{aligned} \qquad (5)$$

III. Timing and Frequency Offset Estimation

In this section, two estimators for carrier frequency and symbol timing recovery are described. Similar to [1] and [4], the MSK system with pilot sequence pattern 101010... is considered in the paper. In the following, the quasistatic channel distortion is assumed, i.e. $\alpha_{k,i} e^{j\theta_{k,i}} = \alpha e^{j\theta}$.

A. Estimator for Frequency Offset

It is observed that for the received pilot sequence, the information-bearing phase $\phi(t)$ is periodic with a period of $2T$. By taking advantage of the periodic property of $\phi(t)$, the phase of $z_{k,i} z^*_{k-m,i}$ can be used to recover the frequency offset when m is even. Here, we define a second order m-lag correlation function

$$C_m(i) = E\{z_{k,i} z^*_{k-m,i}\} \qquad (6)$$

where $E\{\cdot\}$ is the expectation operation, and $0 \le i < N$. When m is even, (6) can be written as

$$C_m(i) = \alpha^2 e^{j2m\pi f_\Delta T} \qquad (7)$$

It indicates that frequency offset can be estimated independent of the estimated symbol timing.

In practice, the expectation $C_m(i)$ is achieved by averaging the samples $z_{k,i} z^*_{k-m,i}$ over the length of the pilot symbol L. Due to the limited length of the pilot symbol, the estimated correlation function $C_m(i)$, denoted as $\hat{C}_m(i)$, can be written as

$$\hat{C}_m(i) = \frac{1}{L-m} \sum_{k=m+1}^{L} \left(z_{k,i} z^*_{k-m,i}\right) \qquad (8)$$

Therefore, the estimated carrier frequency offset is obtained by

$$\hat{f}_\Delta = \frac{1}{2\pi m T} \arg \sum_{i=0}^{N-1} \left(\hat{C}_m(i)\right) \qquad (9)$$

Since m constrains the range of frequency offset, i.e. $|2\pi m f_\Delta T| \le \pi$, to maximize the range of frequency offset, m is set to 2. The estimated frequency offset is used for timing estimation.

B. Estimator for Timing

To estimate the timing error, the following m-lag fourth-order nonlinear transformation of the sampled data is chosen

$$R_m(i) = E\{\left(z_{k,i} z^*_{k-1,i}\right)\left(z_{k-m,i} z^*_{k-m-1,i}\right)\} \qquad (10)$$

It can be further written as

$$R_m(i) = g_m(i) e^{j4\pi f_\Delta T} \qquad (11)$$

where $g_m(i)$ is a carrier frequency offset free term.

$$g_m(i) = \begin{cases} A & m \text{ is odd and } m > 1 \\ -B\cos(2\pi(\varepsilon - \frac{i}{N})) & m \text{ is non-negative even} \end{cases} \qquad (12)$$

where A and B are positive. From (12) it is found that with knowledge of the estimated carrier frequency offset $\hat{f}_\Delta$, the information of the timing error ε can be extracted from $g_m(i)$ where m is non-negative even integer. Due to the limited length of the pilot symbols, the fourth-order expectation $R_m(i)$ is performed by averaging the samples in practice, i.e.,

$$\hat{R}_m(i) = \frac{1}{L-m-1} \sum_{k=m+2}^{L} \left(z_{k,i} z^*_{k-1,i}\right)\left(z_{k-m,i} z^*_{k-m-1,i}\right) \qquad (13)$$

The estimated timing error $\hat{\varepsilon}$ can be obtained with the estimated frequency-offset $\hat{f}_\Delta$ by

$$\hat{\varepsilon} = \frac{1}{2\pi} \arg\left(\sum_{index=0}^{N-1} -\mathrm{sgn}_{index} FR_m(index) e^{-j4\pi \hat{f}_\Delta T} \right). \qquad (14)$$

where sgn_{index} is

$$\mathrm{sgn}_{index} = \begin{cases} 1 & index = 1 \\ -1 & index = N-1 \\ 0 & otherwise \end{cases} \qquad (15)$$

and $FR_m(n)$ is the discrete Fourier transformation of $\hat{R}_m(i)$.

C. Analytical Evaluation

Now, we compute the mean and variance of the estimators from (9) and (14).

Mean. The mean of the estimated frequency offset $\hat{f}_\Delta$ is

$$E\left[\hat{f}_\Delta\right] = E\left[\frac{1}{4\pi T}\arg\sum_{i=0}^{N-1}\hat{C}_2(i)\right] \tag{16}$$

with assumption that for small variance of the estimates, we can linearize the arg-operation, (16) can be written as

$$\begin{aligned} E\left[\hat{f}_\Delta\right] &\approx \frac{1}{4\pi T}\arg E\left[\sum_{i=0}^{N-1}\hat{C}_2(i)\right] \\ &= \frac{1}{4\pi T}\arg(N\alpha^2 e^{j4\pi f_\Delta T}) \\ &= f_\Delta \end{aligned} \tag{17}$$

Similarly, we obtain $E[\hat{\varepsilon}] \approx \hat{\varepsilon}$.

Variance. For simplicity, and without loss of generality, the frequency offset is assumed to be zero.

$$\begin{aligned} \operatorname{var}\{\hat{f}_\Delta\} &= E\{\hat{f}_\Delta^2\} \\ &= \left(\frac{1}{4\pi T}\right)^2 E\left\{\left(\arg\sum_{i=0}^{N-1}\hat{C}_2(i)\right)^2\right\} \\ &\approx \left(\frac{1}{4\pi T}\right)^2 \frac{E\left\{\left(\operatorname{Im}\sum_{i=0}^{N-1}\hat{C}_2(i)\right)^2\right\}}{\left(E\left(\operatorname{Re}\sum_{i=0}^{N-1}\hat{C}_2(i)\right)\right)^2} \\ &= \left(\frac{1}{4\pi T}\right)^2 \frac{1}{(L-2)N}\left(\frac{N_0^2}{2\alpha^4}+\frac{N_0}{\alpha^2}\right) \end{aligned} \tag{18}$$

To calculate the variance of the estimated timing error, we assume $\varepsilon = 0$ and $\hat{f}_\Delta = f_\Delta$ to simplify the expression. Then

$$\begin{aligned} \operatorname{var}(\hat{\varepsilon}) &= E\{\hat{\varepsilon}^2\} \\ &= \left(\frac{1}{2\pi}\right)^2 E\{(\arg X)^2\} \\ &\approx \left(\frac{1}{2\pi}\right)^2 \frac{E\{(\operatorname{Im} X)^2\}}{E\{(\operatorname{Re} X)^2\}} \end{aligned} \tag{19}$$

where

$$X = \sum_{i=0}^{N-1}\frac{1}{L-m-1}\sum_{k=m+2}^{L}\left(z_{k,i}z_{k-1,i}^*\right)\left(z_{k-m,i}z_{k-m-1,i}^*\right)e^{-j4\pi f_\Delta T}\left(e^{-j2\pi(N-1)i/N} - e^{-j2\pi i/N}\right)$$

$E\{(\operatorname{Im} X)^2\}$ can be evaluated by numerical method or the efficient semi-analytical method in [4] for the special oversampling ratio case, i.e. $N = 4$.

IV. Simulation Results

In this section, performance of the proposed algorithm is investigated on the AWGN and Rayleigh fading channels by means of Monte Carlo simulation. Unless indicated otherwise, the simulation system is as follows: the receiver filter is an ideal lowpass filter with a bandwidth covering the signal bandwidth. The symbol period T is $10^{-6}s$, N is set to 4, i.e. the sampling frequency is $4MHz$, and the length of the pilot symbol is 16. For each case with different frequency offset and timing error, 1000 Monte Carlo trials were conducted for each SNR value.

A. AWGN channel

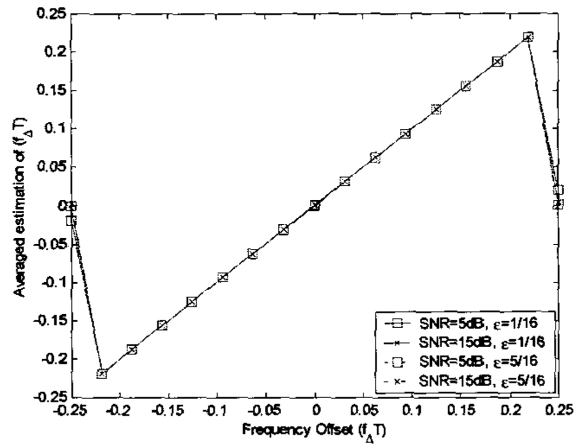

Fig.2 Average estimated frequency offset versus $f_\Delta T$

Synchronization performance in the AWGN channel is first investigated. Fig.2 shows the average estimated frequency versus normalized frequency offset with different timing errors when the SNR is 5dB and 15dB. It is apparent that the estimates are almost identical under different conditions. The proposed estimator for frequency offset has little relationship with the timing error as indicated in (7) and the estimator can work well in low SNR cases. It is also seen that within the range $|f_\Delta T| < 0.22$, the estimates are the same as the ideal case $E\{f_\Delta T\} = f_\Delta T$.

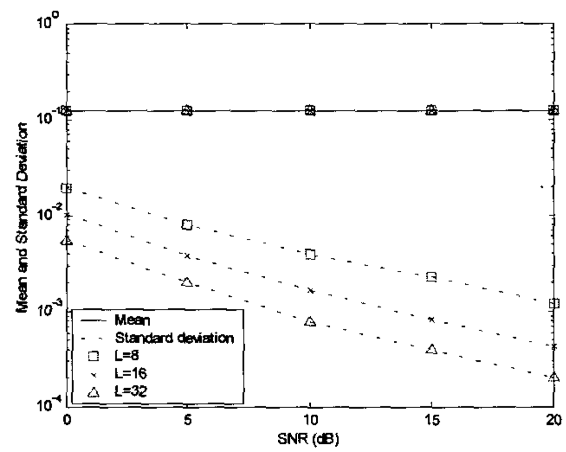

Fig.3.Mean and standard deviation of estimated $f_\Delta T$ vs. SNR ($f_\Delta T$ =1/8)

In Fig.3, the mean and standard derivation of the estimated frequency offset is plotted against SNR. The mean for three different pilot symbol lengths coincide which indicates the estimator for freqency offset is consistent. It can also be observed that when the length of pilot sequence is equal to or greater than 16, the standard deviation can reach below 5×10^{-3}, even at low SNR case of about 5dB.

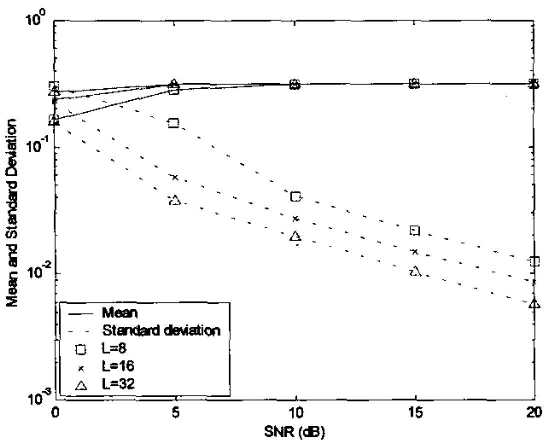

Fig.4. Mean and standard deviation of estimated symbol timing vs. SNR (ε=5/16)

Fig.4 shows the mean and standard derivation of the timing estimation versus SNR for different length of pilot sequence. It can be seen that when SNR is high (>10dB), the mean of the timing estimates concide for the 3 cases. At low SNR, the estimation error for short pilot length is higher probably due to the effect of noise as well as the linear approximation of small variance in (17).

Fig 5 shows the timing estimation of the proposed algorithm when m equals to 2 and 0 as well as the algorithm proposed in [1]. Here the pilot symbol length is set to 16. It is apperant that the normalized standard derivation of the proposed algorithm is much better than that of the algorithm in [1], especially when the SNR is below 10dB.

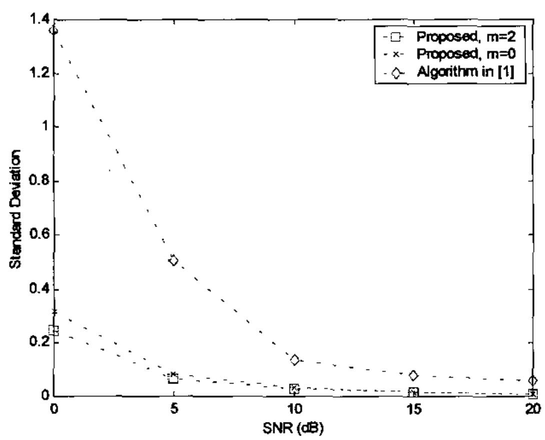

Fig 5. Performance comparison of timing estimaton algorithms

B. Rayleigh fading channel

In this section, simulation results in Rayleigh fading channel are presented.

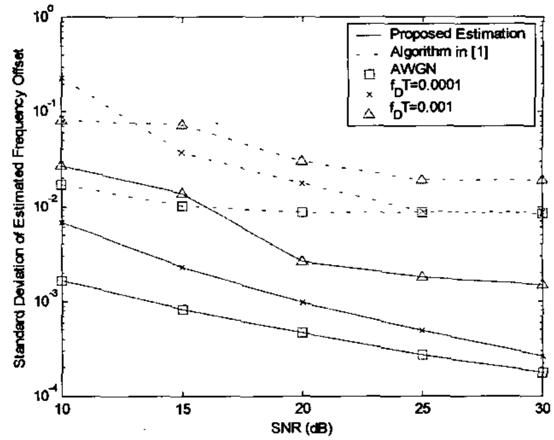

Fig 6. Performance of frequency offset estimator in Rayleigh fading channel

In Fig.6, performance of the estimated frequency offset is compared with that obtained with the algorithm in [1] under three different fading environments, i.e. non-fading, fading at normalized Doppler frequency $f_D T = 0.0001$ and 0.001, respectively. It can be seen that even in very slow fading channel, the standard derivation for the estimated frequency offset degrades due to the fluctuation of fading gain and phase, and larger normalized Doppler frequency leads to more serious performance degradation. It is also clearly demonstrated that the proposed algorithm performs better than that of [1].

In Fig.7, the standard deviation of the symbol timing estimates in Rayleigh fading channel is plotted. Similar observation can be made as in the case of frequency offset estimation shown in Fig.6.

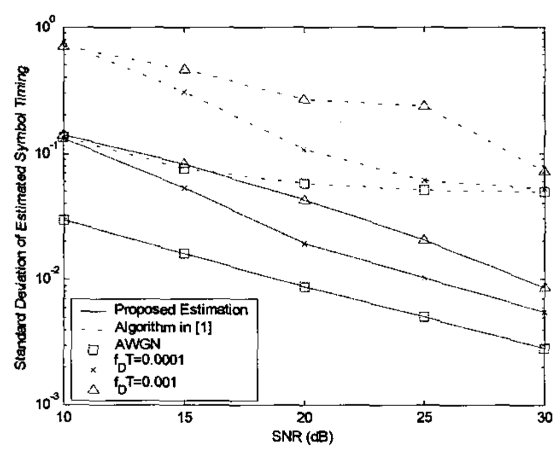

Fig 7. Performance of estimated symbol timing in Rayleigh fading channel

V. Conclusion

In the paper, a novel data-aided estimation scheme for both frequency offset and timing error with MSK signals has been proposed. The estimations are based on second-order and fourth-order statistical properties of the MSK signals. Simulation results have shown that the estimator for frequency offset and estimator for timing error achieves good performance in both AWGN and slow Rayleigh fading channel, and it outperforms substantially a previously published data-aided schems.

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