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Indirect Adaptive Control for Systems with an Unknown Dead Zone

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Abstract

Dead-zone inverse methods have been used in adaptive control schemes to compensate for systems with an unknown dead zone. The problem with these techniques is that steady state error may still exist. It is shown in this paper that controller with integrating action can be used to remove steady state error arising from the unknown dead zone. By treating the effect of an unknown dead zone as a bounded disturbance being injected into the system, a plant parametrization that is linear in a set of unknown parameters is developed and the estimation algorithm is proposed. A novel feature of the adaptive controller proposed here is the integrating action in the controller. Stability analysis shows that the adaptive scheme ensures boundedness of all closed-loop signals and eliminates tracking errors. As illustrated in a simulation example, the proposed adaptive controller is simple to implement and accurate tracking can be achieved.

Key words: unknown dead-zone, adaptive controller with integrating action, steady state error and boundedness

1. Introduction

Dead zone is a common nonlinearity in actuators such as hydraulic servo-valves. The difficulty in compensating for dead zone is that its parameters are generally unknown and may vary with time. However, it is well known that without suitable compensation, the response of a system designed assuming the system is linear can deteriorate rapidly, leading to steady state errors, oscillatory response or even instability. To compensate for dead zone, a direct adaptive control has been analyzed by Recker *et al.* [1]. A recursively updated dead zone inverse is introduced in front of the dead zone to minimize the effect of the dead zone. Tao and Kokotovic [2, 3] extended the method from full state measurement to a single output measurement. Recently, Recker *et al.* [4] extended the direct approach to an indirect one for discrete systems. The limitation of using dead zone inverse is steady state errors can still arise. This is because the estimated dead zone parameters are not unique, and consequently may not converge to their true values. As the dead zone cannot be eliminated completely using the estimated dead zone inverse, steady state errors may exist, as discussed later in this paper.

A general compensator for known systems with unknown dead zone is proposed in [5]. The motivation for the general dead zone compensation is that the difference between the controller output and the dead zone actuator output can be considered as a disturbance being injected into the system. Dead zone compensation is then achieved by adding a term involving the disturbance to the controller. With a suitable choice of the compensator, steady state error can be eliminated and the transient response and the stability of the compensated system be improved.

In this paper, the approach proposed in [5] is extended to adaptive control. Instead of using the dead zone inverse, a controller with integrating action is proposed to eliminate the steady state error arising from the dead zone. The approach proposed here not only has the stability properties discussed in [1-4] but also eliminates the steady state error, which is not always possible in the other approaches.

This paper is organized as follows. In Section 2, the control problem is formulated. The linear parametrization of the plant and a parameter estimation algorithm are discussed in Section 3. The computation of the controller parameters using the estimated plant parameters and the stability analysis of the system are given in Section 4. An example illustrating the results is given in Section 5.

2. Problem Formulation

Consider a system consisting of a linear time invariant plant $\frac{B_1(s)}{A_1(s)}$ and an unknown dead zone, as shown in Fig.1. The dead zone can be expressed by two parts: deadzone given by

$$v' = \begin{cases} v(t) - b_r & v(t) \geq b_r \\ 0 & b_l < v(t) < b_r \\ v(t) - b_l & v(t) \leq b_l \end{cases} \tag{2.1a}$$

and gain

$$k_d = \begin{cases} m_r & v(t) \geq 0 \\ m_l & v(t) < 0 \end{cases} \tag{2.1b}$$

where $m_l,\ m_r > 0$. (2.1a) implies control cannot be fully implemented by the actuator and can be interpreted as introducing a nonlinear disturbance δ' in the system, where δ' is defined as,

$$\delta' = v - v' = \begin{cases} -b_r & v(t) \geq b_r \\ -v(t) & b_r < v(t) < b_r \\ -b_l & v(t) \leq b_l \end{cases} \tag{2.2}$$

The output of the system is now given by,

$$y = k_d(v)\frac{B_1(s)}{A_1(s)}(v + \delta') = \frac{B(s)}{A(s)}(v + \delta') \tag{2.3}$$

where $B(s) = k_d B_1(s)$, $A(s) = A_1(s)$ and s is Laplace transfer variable. It should be noted that if $m_l \neq m_r$, the parameters of $B(s)$ is not constant at $v(t) = 0$ point. Suppose the system satisfies the following assumptions,

- S1 The piecewise linear nonlinearity is in the span of the control;
- S2 Except point $v(t) = 0$, $B(s)/A(s)$ is linear with unknown coefficients but known number of poles n and zeros m;
- S3 The zeros and the poles of $B(s)/A(s)$ are stable;
- S4 $A(s)$ and $B(s)$ are coprime, and $A(s)$ is monic.

The control objective is to design a feedback control $v(t)$ such that the closed-loop signals are bounded and the plant output $y(t)$ tracks the output y_m of the reference model,

$$y_m = W_m(s)u_c = \frac{B_m(s)}{A_m(s)}u_c \tag{2.4}$$

where $A_m = (s + a_1)(s + a_2)A_{ml}$ is monic polynomial of degree $(n+2)$ whose roots are the desired poles of the closed-loop system; B_m is a polynomial with stable roots of degree m; u_c is input.

3. Plant Parameter Estimation

To estimate the plant parameters in (2.3), a parametric model is first derived as following. Because of the specific characteristic of $\delta'(t)$, i.e., it is a constant when $|v(t)| \geq \max\{b_r, -b_l\}$, $\frac{s}{\Lambda(s)}$ is introduced in (2.3) and the linear parametrization can be obtained as,

$$y_f = \varphi^T(t)\,\theta + \eta \tag{3.1}$$

where

$$y_f = \frac{s^{n+1}}{\Lambda(s)}[y] \qquad \eta = B(s)\left(\frac{s}{\Lambda(s)}\delta'\right) \tag{3.1a}$$

$$\varphi^T = \left[-\frac{s^n}{\Lambda(s)}[y] \cdots -\frac{s}{\Lambda(s)}[y] \quad \frac{s^{m+1}}{\Lambda(s)}[u] \cdots \frac{s}{\Lambda(s)}[u]\right] \tag{3.1b}$$

and $\Lambda(s)$ is a Hurwitz polynomial of degree $(n+1)$. The advantage of introducing $s/\Lambda(s)$ is that it has a differential effect and

$$\frac{d\delta'}{dt} = \begin{cases} -\frac{dv}{dt} & b_l < v(t) < b_r \\ 0 & others \end{cases} \tag{3.2}$$

Hence, the influence of $\delta'(t)$ on parameter estimation is eliminated when $v \notin (b_l,\ b_r)$. In the following, supposed $\bar{b}$ is the known upper bounds of b_r, $-b_l$, θ is estimated only when $|v(t)| \geq \bar{b}$. As shown in assumption S2 in section 2, θ is constant in the adaptive duration.

Consider the cost function

$$J(\theta, t) = \frac{1}{2}e^{-\alpha t}(\theta - \theta_0)^T Q_0(\theta - \theta_0) + \frac{1}{2}\int_0^t e^{-\alpha(t-\iota)}\left[\frac{(y_f(\iota) - \phi^T(\iota)\theta)^2}{m^2(\iota)} + \omega(\iota)\theta^T(t)\theta(\iota)\right]d\varsigma \tag{3.3}$$

where $Q_0 = Q_0^T > 0$, $\alpha \geq 0$ and $\theta_0 = \theta(0)$; m is designed as $m^2 = c_1 + c_2 n_s^2$ with c_1, $c_2 > 0$ such that $\frac{\varphi}{m}$, $\frac{\eta}{m} \in L_\infty$. For the plant parametrization of (3.1), the switching-σ least-squares law in [6] with the dead-zone modification is as follows,

$$\frac{dP}{dt} = \begin{cases} (\alpha P - P\frac{\varphi\varphi^T}{m^2}P)\bar{f}(v) & \text{if } |P(t)| \leq R_0 \\ 0 & \text{otherwise} \end{cases} \tag{3.4a}$$

where $\bar{f}(v) = \begin{cases} 1 & |v(t)| \geq \bar{b} \\ 0 & \text{others} \end{cases}$, $P(0) = Q_0^{-1} \leq R_0$ and R_0 is a constant that serves as an upper bound for $P(t)$.

$$\frac{d\theta}{dt} = -P(\varphi\epsilon + \omega\theta)\bar{f}(v) \tag{3.4b}$$

where

$$\epsilon = \frac{\varphi^T(t)\theta(t) - y_f(t)}{m^2(t)} \tag{3.4c}$$

$$\omega = \begin{cases} 0 & \text{if } \|\theta\| \leq M_0 \\ \left(\frac{\|\theta\|}{M_0} - 1\right)^l \sigma_0 & \text{if } M_0 < \|\theta\| \leq 2M_0 \\ \sigma_0 & \text{if } \|\theta\| > 2M_0 \end{cases} \tag{3.4d}$$

with the design constant σ_0 and $M_0 > \|\theta^*\|$, and l any finite positive integer. The modified least-squares algorithm has the following properties:

Theorem 1 *The switching-σ least-squares law with dead zone modification (3.4) for the system with unknown dead zone guarantees:*

(i) ϵ, ϵn_s, θ, $\dot{\theta} \in L_\infty$;

(ii) ϵ, ϵn_s, $\dot{\theta} \in L_2$; *and*

(iii) $\dot{\theta} \in L_1$, $\lim_{t\to\infty}\theta = \bar{\theta}$.

Proof: Consider the function,

$$V(\phi, t) = \frac{\phi^T P^{-1}\phi}{2} \tag{3.5}$$

where $\phi = \theta - \theta^*$. Following the proof of the switching-σ least-squares law in [6] and considering $\eta = 0$ in the parameter adaption, (i) and (ii) can be obtained directly. Furthermore, the time derivative $\dot{V}$ along the solution of (3.4) is,

$$\frac{dV}{dt} \leq -(\frac{\epsilon^2 m^2}{2} + \omega\phi^T\theta)\bar{f}(v) \tag{3.6}$$

where $\omega\phi^T\theta = \omega(\|\theta\|^2 - \theta^{*T}\theta) \geq \omega\|\theta\|(\|\theta\| - M_0 + M_0 - \|\theta\|)$ from (3.4b). Since $\omega(\|\theta\| - M_0) \geq 0$ and $M_0 > \|\theta^*\|$, it follows that $\omega\phi^T\theta \geq 0$. Hence,

$$\frac{dV}{dt} \leq -\frac{1}{2}(\epsilon^2 m^2 + \omega\phi^T\theta)\bar{f}(v) = -\frac{1}{2}|\phi^T|\,|\epsilon\varphi + \omega\theta|\bar{f}(v) \tag{3.7}$$

Since ϕ, $V \in L_\infty$, $|\epsilon\varphi + \omega\theta|\bar{f}(v) \in L_1$. Therefore from (3.4), $\|\dot{\theta}\| \in L_1$, which implies that $\int_0^\infty \dot{\theta}dt$ exists and $\lim_{t\to\infty}\theta = \bar{\theta}$. □

4. Controller Design and Stability Analysis

A block diagram of our adaptive system is shown in Fig. 2, where the controller can be expressed as,

$$v = \frac{\hat{T}(s)}{\hat{R}(s)}u_c - \frac{\hat{S}(s)}{\hat{R}(s)}y \tag{4.1}$$

or

$$v = \frac{\Lambda_p - \hat{R}}{\Lambda_p}v + \frac{\hat{T}u_c - \hat{S}y}{\Lambda_p} \tag{4.2}$$

where $\hat{T}(s)$, $\hat{R}(s)$, $\hat{S}(s)$ are calculated from the estimated parameters of $A(s)$ and $B(s)$, as discussed in the following; Λ_p is monic, Hurwitz polynomial with degree of $\deg(\hat{R})$. The output can be obtained from Fig.2 as,

$$y = \frac{B\hat{T}}{B\hat{S} + A\hat{R}}u_c + \frac{B\hat{R}}{B\hat{S} + A\hat{R}}\delta' \tag{4.3}$$

It can be seen that even if the system is stable and $\lim_{t\to\infty} v(t) = \text{constant}$, the steady state error may not be eliminated since $\lim_{t\to\infty}\delta' \neq 0$ may not be guaranteed. To solve the problem, Theorem 2 is proposed.

Theorem 2 *For stable adaptive control systems satisfying* $\lim_{t\to\infty} v(t) = \text{constant}$, *if dead zone exists in the actuator, the sufficient condition to eliminate the steady state error caused by the dead zone is that controller has integrating action, i.e.,* $\hat{R}(0) = 0$.

Proof: Since $\lim_{t\to\infty} v(t) = \text{constant}$, we have

$\lim_{t\to\infty} \delta' = \text{constant}$ from (2.2). As $\hat{R}(0)=0$, $d\delta'/dt$ can be obtained from (4.3), giving $d\delta'/dt$ and the second term in the right of (4.3) approaching zero as $t\to\infty$. □

In the following, how to calculate the controller with integrating action is discussed. To construct a controller that achieves model matching, the following Diophantine Equation must be solved,

$$\hat{A}\hat{R}_1 + \hat{B}\hat{S}_1 = A_{ml}A_0 \tag{4.4}$$

where A_0 is an observer polynomial and all its roots are in the left half of s-plane, and A_{ml} is defined in (2.4). If $\hat{S}_1$, $\hat{R}_1$ have degree of $\deg A - 1$ and $\deg A_0$ respectively, and $\hat{R}_1$ is monic, then there always exists a unique solution $\hat{R}_1$, $\hat{S}_1$. Furthermore, if A_0 is chosen to satisfy $\deg A_0 > \deg A - 1$, then $\hat{S}_1/\hat{R}_1$ is strictly proper for all estimates of the plant poles. The controller with integrating action is calculated as,

$$\hat{R}(s) = (s+a_1)(s+a_2)\hat{R}_1 - \frac{a_1 a_2 \hat{R}_1(0)}{\hat{B}(0)}\hat{B}(s) \tag{4.5}$$

$$\hat{S}(s) = (s+a_1)(s+a_2)\hat{S}_1 + \frac{a_1 a_2 \hat{R}_1(0)}{\hat{B}(0)}\hat{A}(s) \tag{4.6}$$

$$\hat{T} = A_0 \hat{B}_m' \tag{4.7}$$

where $(s+a_1)(s+a_2)$ is defined in (2.4) and $B_m = \hat{B}B_m'$. From (4.4)-(4.7), the following results can be obtained.

Remark If $\hat{S}_1$, $\hat{R}_1$ are chosen to satisfy (4.4) with degree of $\deg A - 1$ and $\deg A_0$ and $\deg A_0 > \deg A - 1$, then controller $\hat{T}$, $\hat{R}$ and $\hat{S}$ calculated from (4.5)-(4.7) guarantees that,

R1 The controller has integrating action;

R2 $\hat{T}$, $\hat{R}$ and $\hat{S}$ satisfy $\hat{A}\hat{R} + \hat{B}\hat{S} = (s+a_1)(s+a_2)A_{ml}A_0$ and $\frac{\hat{T}(0)}{\hat{S}(0)} = \frac{B_m(0)}{A_m(0)}$; and

R3 $\frac{\hat{A}\hat{T}s}{A_0A_{ml}(s+a_1)(s+a_2)}$ and $\frac{\hat{S}\Lambda}{A_0A_{ml}(s+a_1)(s+a_2)}$ are strictly proper rational and analytic in $Re\{s\} \geq 0$.

To analyze the properties of the adaptive control system, the following Lemmas are needed.

Lemma 4.1 *A strictly proper rational transfer function $H(s)$ is analytic in $R_e\{s\} \geq 0$ if and only if $h \in L_1$, where $h(t)$ is the system's impulse transfer function and $H(s)$ is the Laplace transfer function of $h(t)$.*

Lemma 4.2 *If $h \in L_{1,e}$ then $\|(h*u)_t\|_p \leq \|h_t\|_1 \|u_t\|_p$, where $P \in [1,\infty]$.*

The system designed has the following properties:

Theorem 3: *Given the modified least-squares update law (3.4) and the controller (4.5)-(4.7), applied to the dead zone plant, y(t) and v(t) satisfy,*

$$y(t),\ v(t) \in L_\infty$$

and tracking error $e_m = y - y_m$ approaches zero as $t\to\infty$.

Proof: Let us define $v_f = \frac{s}{\Lambda_p}v$, $y_f = \frac{s}{\Lambda_p}y$ and rewrite (3.4c) and (4.1) as

$$\hat{A}\Lambda_q y_f - \hat{B}\Lambda_q v_f = \epsilon m^2 \tag{4.8}$$

$$\hat{R}(s)v_f + \hat{S}(s)y_f = y_{ml} \tag{4.9}$$

where $\Lambda_p = \Lambda\Lambda_q$ and $y_{ml} = \frac{\hat{T}s}{\Lambda_p}u_c \in L_\infty$. The forms (4.8) and (4.9) are exactly the same as those of linear system. Following exactly the same steps given in [7], and considering Theorem 1, we can show that $\epsilon m^2 \in L_\infty$.

Define $\bar{v} = \frac{s}{\Lambda}v$, $\bar{y} = \frac{s}{\Lambda}y$ and write (3.4c) as

$$\bar{y}^{(n)} + a_{n-1}\bar{y}^{(n-1)} + \dots + a_0\bar{y} = b_m\bar{v}^{(m)} + \dots + b_0\bar{v} - \epsilon m^2 \tag{4.10}$$

giving,

$$\bar{y} = \frac{\hat{B}(s)}{\hat{A}(s)}\bar{v} - \frac{1}{\hat{A}(s)}\epsilon m^2 \tag{4.11}$$

Filtering v, u_c and y in (4.1) by $s/\Lambda(s)$, and substituting $\bar{y}$ in (4.11) into (4.1),

$$[\hat{A}(s)\hat{R}(s) + \hat{S}(s)\hat{B}(s)]\,\bar{v} = \hat{A}(s)\hat{T}(s)\bar{u}_c + \hat{S}(s)\,\epsilon m^2 \tag{4.12}$$

From Remark R2, (4.12) can be rewritten as

$$\bar{v} = \frac{\hat{A}\hat{T}}{A_0 A_{ml}(s+a_1)(s+a_2)}\bar{u}_c + \frac{\hat{S}}{A_0 A_{ml}(s+a_1)(s+a_2)}\epsilon m^2 \quad (4.13)$$

Multiplying $\Lambda(s)$ to (4.13),

$$sv = \frac{\hat{A}\hat{T}s}{A_0 A_{ml}(s+a_1)(s+a_2)}u_c + \frac{\hat{S}\Lambda}{A_0 A_{ml}(s+a_1)(s+a_2)}\epsilon m^2 \quad (4.14)$$

because of the boundedness of u_c and ϵm^2 and Remark R3, $dv/dt \in L_\infty$ according to Lemma 4.1 and 4.2. Dividing s to (4.14), for same reason $v \in L_\infty$. This in turn guarantees that $y \in L_\infty$ from (2.2) and (2.3).

As dv/dt, $v \in L_\infty$, $\lim_{t\to\infty} \frac{dv}{dt} = 0$ according to Barbalat Theorem, i.e., $\lim_{t\to\infty} v(t) = \text{constant}$. Furthermore, δ' and $y(t)$ approach to constant from (2.2) and (2.3), respectively. Applying final value theorem to (4.3) and considering Theorem 2 and Remark R2, $\lim_{t\to\infty} y(t) = \lim_{t\to\infty} y_m(t)$. □

A procedure to calculate the adaptive control is as below.

Step 1: Estimate the parameters in (3.1) by the modified least-squares approach (3.4);

Step 2: From the estimates of Step 1, compute the controller using (4.4)-(4.7);

Step 3: calculate the control signal from (4.2).

5. Example

Consider a system consisting of a linear time invariant plant $\frac{B_1(s)}{A_1(s)} = \frac{b_0}{s^2 + a_1 s + a_0}$ with $a_0 = 0$, $a_1 = b_0 = 1$, and an unknown dead zone with the following parameters,

$$b_r = 1.5, \quad b_l = -1.2, \quad k_d(v) = \begin{cases} 2 & v \geq 0 \\ 3.6 & v < 0 \end{cases}.$$

The simulation result is presented for the following controller,

$$W_m = \frac{50}{(s^2 + 1.4s + 1)(s^2 + 15s + 50)}.$$

Assume the known upper bounds of b_r and b_l are $\bar{b} = 2$. To estimate the parameters of $\frac{B(s)}{A(s)} = k_d(v)\frac{B_1(s)}{A_1(s)}$, $s/\Lambda(s)$ and the initial values of P and θ are chosen as $\frac{s}{s^3 + 6.4s^2 + 8s + 5}$, $10^3 I_3$ and $[3\ 5\ 8]^T$, respectively. To calculate the controller with the integral action, $A_0(s) = (s+2)^2$ is chosen. The simulation results for the system with and without dead zone are shown in Fig.3 for comparison. It can be seen that the accurate tracking is achieved.

6. Conclusions

In this paper, an indirect adaptive control scheme is proposed for a linear system with an unknown dead zone. Representing the effects of an unknown dead zone by a bounded disturbance, a plant parametrization that is linear in a set of unknown parameters is developed and the estimation algorithm is proposed. A novel feature of the proposed adaptive controller is its integral action. It is shown that the integral action is the sufficient condition to eliminate the steady state error caused by dead zone. Stability analysis shows that the adaptive scheme ensures boundedness of all closed-loop signals and eliminates tracking error. Simulation shows that our adaptive controller is simple to implement and accurate tracking can be achieved.

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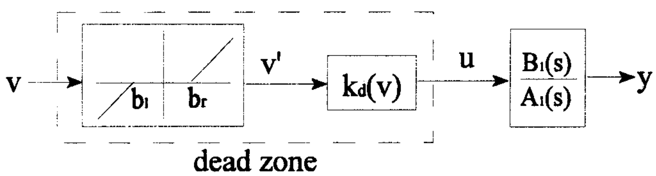

Figure 1 A system with unknown dead zone

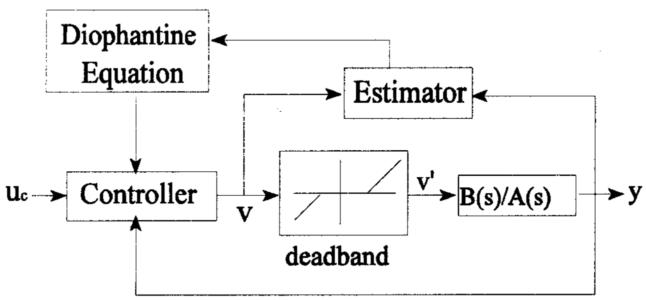

Figure 2 The compensated system

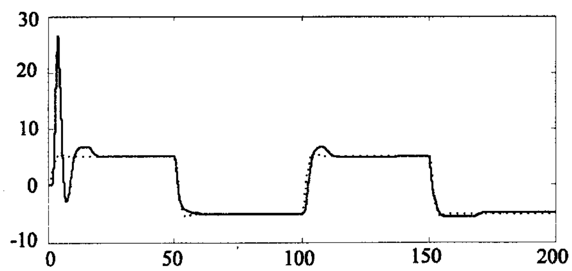

Figure 3 System responses
... system without dead zone _ system with dead zone