Dynamic Response of Multiple Coplanar Interface Cracks Between Two Dissimilar Piezoelectric Materials

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ABSTRACT

The linear piezoelectricity theory is applied to investigate the dynamic response of coplanar interface cracks between two dissimilar piezoelectric materials subjected to the mechanical and electrical impacts. The number of cracks is arbitrary, and the interface cracks are assumed to be permeable for electric field. Integral transforms and dislocation density function are employed to reduce the problem to Cauchy singular integral equations. Numerical examples are given to show the effects of crack relative position and material property parameters on the variations of dynamic energy release rate.

1. INTRODUCTION

Due to the intrinsic coupling characteristics between electric and elastic behaviors, piezoelectric materials have been used widely. Studies on electroelastic problem of a piezoelectric material with cracks in the framework of the theory of piezoelectricity were initiated by Parton[1] and Deeg[2]. Since their pioneering works, the problem of the determination of electroelastic field under different boundary conditions has been investigated by a number of researchers.

The dynamic response of crack problems in a piezoelectric material under various time-dependent loads is of great importance in some practical applications and has recently received much attention[3,4]. However, for interface crack problem, most studies are concentrated on the case of one mode-III crack with electric impermeable crack surface condition[5].

In fact, for mode-III crack, electric permeable crack surface condition is perhaps more appropriate than electric impermeable condition. In this paper, studied is the transient response of coplanar interface cracks between two dissimilar piezoelectric materials subjected to anti-plane mechanical and electrical impacts. The number of coplanar crack is arbitrary, and crack surfaces are assumed to be permeable for electric field. Integral transforms and dislocation density function are used to reduce the problem to singular integral equations, which are numerically solved.

2. STATEMENT OF THE PROBLEM

Consider n mode-III Griffith interface cracks between two bonded transversely isotropic piezoelectric materials, with their basal planes perpendicular to the z axis as shown in Fig. 1. The x coordinates of the kth crack tips are set to be a_k and b_k $(k = 1 \sim n)$. The antiplane shear impact and the electric displacement impact are imposed on the crack surfaces at $t = 0$.

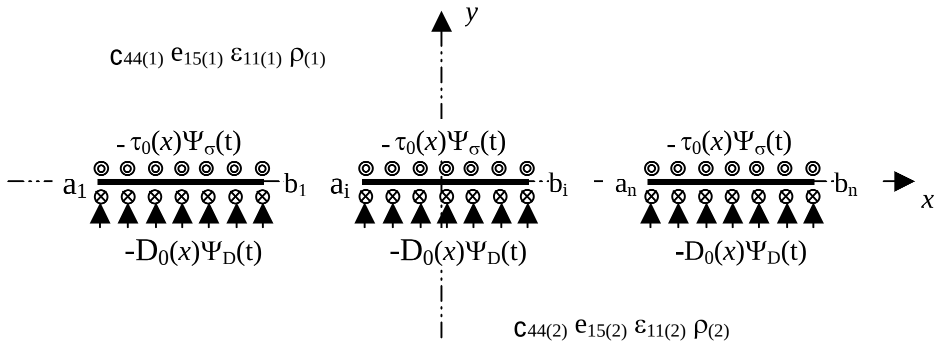

Fig.1. Two dissimilar piezoelectric materials with multiple coplanar interface cracks under anti-plane mechanical impact and in-plane electrical impact

The governing equations of the dynamic antiplane problem are

$$c_{44(j)}\nabla^2 w_{(j)} + e_{15(j)}\nabla^2 \phi_{(j)} = \rho_{(j)}\frac{\partial^2 w_{(j)}}{\partial t^2} \qquad j = 1,2 \tag{1}$$

$$e_{15(j)}\nabla^2 w_{(j)} - \varepsilon_{11(j)}\nabla^2 \phi_{(j)} = 0 \qquad j = 1,2 \tag{2}$$

where ∇^2 is the two-dimensional Laplace operator. $w_{(j)}(x, y, t)$ and $\phi_{(j)}(x, y, t)$ are the non-zero elastic displacements and electric potentials, respectively. The quantities with the subscript (j) denote the corresponding quantities in the upper and lower materials, respectively. $c_{44(j)}$, $\varepsilon_{11(j)}$, $e_{15(j)}$ are the elastic, dielectric and piezoelectric constants, respectively.

The boundary conditions for electric permeable interface cracks can be written as

$$\sigma_{zy(1)}(x,0,t) = \sigma_{zy(2)}(x,0,t) = -\tau_0(x)\Psi_\sigma(t) \qquad x \in \bigcup_{k=1}^{n}(a_k, b_k) \tag{3}$$

$$\sigma_{zy(1)}(x,0,t) = \sigma_{zy(2)}(x,0,t) \qquad -\infty < x < +\infty \tag{4}$$

$$\phi_{(1)}(x,0,t) = \phi_{(2)}(x,0,t) \qquad -\infty < x < +\infty \tag{5}$$

$$D_{y(1)}(x,0,t) = D_{y(2)}(x,0,t) \qquad -\infty < x < +\infty \tag{6}$$

$$w_{(1)}(x,0,t) - w_{(2)}(x,0,t) = \Delta w(x,t) \qquad -\infty < x < +\infty \tag{7}$$

where

$$\Delta w(x,t)=\begin{cases}\Delta w_k(x,t)=w_{(1)}(x,0,t)-w_{(2)}(x,0,t) & x\in(a_k,b_k)\ \ k=1,2,\cdots,n\\ 0 & x\notin\bigcup_{k=1}^{n}(a_k,b_k)\end{cases}$$

$\sigma_{zy(j)}$ and $D_{y(j)}$ are stresses and electric displacements, respectively. $\tau_0(x)$ is known function of x, and $\Psi_\sigma(t)$ of t. Δw_k is called dislocation function of the k th crack.

3. THE DERIVATION AND SOLUTION OF SINGULAR INTEGRAL EQUATION

Introducing Laplace and Fourier transforms, the solutions of Eqs.1 and 2 are obtained as

$$w_{(j)}{}^*(x,y,p)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[A_{(j)}(s,p)e^{-\alpha_{(j)}|y|}\right]e^{-isx}ds \qquad j=1,2 \tag{8}$$

$$\phi_{(j)}{}^*(x,y,p)=\frac{e_{15(j)}}{\varepsilon_{11(j)}}w_{(j)}{}^*(x,y,p)+\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[B_{(j)}(s,p)e^{-|s||y|}\right]e^{-isx}ds \qquad j=1,2 \tag{9}$$

where the quantities with superscript asterisk denote the corresponding quantities in Laplace transform domain, $A_{(j)}$, $B_{(j)}$ $(j=1,2)$ are the unknowns to be solved and

$$\alpha_{(j)}=\sqrt{s^2+p^2/c_{T(j)}{}^2}\ ,\ \ c_{T(j)}=\sqrt{\tilde{c}_{44(j)}/\rho_{(j)}}\ ,\ \ \tilde{c}_{44(j)}=c_{44(j)}+e_{15(j)}{}^2/\varepsilon_{11(j)}$$

Substituting Eqs.8 and 9 into the corresponding constitutive relations in Laplace transform domain and using Eqs.4 - 7 yields

$$\begin{bmatrix}-\tilde{c}_{44(1)}\alpha_{(1)} & -\tilde{c}_{44(2)}\alpha_{(2)} & -e_{15(1)}|s| & -e_{15(2)}|s|\\ e_{15(1)}/\varepsilon_{11(1)} & -e_{15(2)}/\varepsilon_{11(2)} & 1 & -1\\ 0 & 0 & \varepsilon_{11(1)}|s| & \varepsilon_{11(2)}|s|\\ 1 & -1 & 0 & 0\end{bmatrix}\begin{Bmatrix}A_{(1)}\\ A_{(2)}\\ B_{(1)}\\ B_{(2)}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ 0\\ \overline{\Delta w^*}(s,p)\end{Bmatrix} \tag{10}$$

in which $\overline{\Delta w^*}$ is the Fourier transform of Δw^*. According to the Cramer's rule, we get

$$A_{(1)}(s)=\Delta_{41}(s,p)\overline{\Delta w^*}(s,p)/\Delta(s,p) \tag{11}$$

$$B_{(1)}(s)=\Delta_{43}(s,p)\overline{\Delta w^*}(s,p)/\Delta(s,p) \tag{12}$$

where $\Delta(s,p)$ is the determinant of the coefficient matrix of equation system 10. $\Delta_{41}(s,p)$ and $\Delta_{43}(s,p)$ are, respectively, the corresponding algebra cofactors when applying the Cramer's rule.

Substituting Eqs.8 and 9 into constitutive relations and using Eqs.11 and 12, we have from Eq.3

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(-\tilde{c}_{44(1)}\alpha_{(1)}\Delta_{41}-e_{15(1)}|s|\Delta_{43}\right)\overline{\Delta w^*}(s,p)/\Delta(s,p)e^{-isx}ds=-\tau_0(x)\Psi_\sigma{}^*(p) \qquad x\in\bigcup_{k=1}^{n}(a_k,b_k) \tag{13}$$

Substituting $\overline{\Delta w^{*}}(s,p)$ for $\Delta w^{*}(v,p)$ and noting that $\Delta(s,p)$, $\Delta_{41}(s,p)$ and $\Delta_{43}(s,p)$ are all even functions of s, we get by by-part integration

$$\frac{1}{\pi}\sum_{k=1}^{n}\int_{a_k}^{b_k}\frac{\gamma_{11}}{v-x}f_k(v,p)dv+\frac{1}{\pi}\sum_{k=1}^{n}\int_{a_k}^{b_k}Q_{11}(v,x,p)f_k(v,p)dv=-\tau_0(x)\Psi_{\sigma}^{*}(p)\qquad x\in\bigcup_{l=1}^{n}(a_l,b_l)\qquad(14)$$

where

$$Q_{11}(v,x,p)=\int_0^{+\infty}\left(\frac{\tilde{c}_{44(1)}\alpha_{(1)}\Delta_{41}+e_{15(1)}|s|\Delta_{43}}{s\Delta(s,p)}-\gamma_{11}\right)\sin s(v-x)ds$$

$$\gamma_{11}=\frac{\tilde{c}_{44(1)}e_{15(2)}{}^{2}\varepsilon_{11(1)}{}^{2}+\tilde{c}_{44(2)}e_{15(1)}{}^{2}\varepsilon_{11(2)}{}^{2}-\tilde{c}_{44(1)}\tilde{c}_{44(2)}\varepsilon_{11(1)}\varepsilon_{11(2)}\left(\varepsilon_{11(1)}+\varepsilon_{11(2)}\right)}{e_{15(2)}{}^{2}\varepsilon_{11(1)}{}^{2}+e_{15(1)}{}^{2}\varepsilon_{11(2)}{}^{2}-\left(\tilde{c}_{44(1)}+\tilde{c}_{44(2)}\right)\varepsilon_{11(1)}\varepsilon_{11(2)}\left(\varepsilon_{11(1)}+\varepsilon_{11(2)}\right)-2e_{15(1)}e_{15(2)}\varepsilon_{11(1)}\varepsilon_{11(2)}}$$

$f_k(v,p)=\dfrac{\partial\Delta w_k{}^{*}(v,p)}{\partial v}$ is called dislocation density function of the k th crack. By setting $c_k=(b_k-a_k)/2$, $d_k=(b_k+a_k)/2$ and applying the substitutions

$$v=c_k\eta+d_k\,,\quad x=c_k\varsigma+d_k$$
$$F_k(\eta,p)=f_k(c_k\eta+d_k,p),\quad \tau_{0,k}(\eta)=\tau_0(c_k\eta+d_k,p)$$
$$\tilde{Q}_{11}^{kl}(\eta,\varsigma,p)=c_kQ_{11}(c_k\eta+d_k,c_l\varsigma+d_l,p)+c_k(1-\delta_{kl})\gamma_{11}/[(c_k\eta+d_k)-(c_l\varsigma+d_l)]$$

Eq.14 can be further converted to the following standard singular integral equation

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\gamma_{11}}{\eta-\varsigma}F_l(\eta,p)d\eta+\frac{1}{\pi}\int_{-1}^{1}\sum_{k=1}^{n}\tilde{Q}_{11}^{kl}(\eta,\varsigma,p)F_k(\eta,p)d\eta=-\tau_{0,l}(\varsigma)\Psi_{\sigma}^{*}(p)\quad|\varsigma|<1\qquad l=1,2,\cdots,n\ (15)$$

The single-valued condition of Eq.15 may be expressed as

$$\int_{-1}^{1}F_k(\eta,p)d\eta=0\qquad k=1,2,\cdots,n\qquad(16)$$

Eq.15 is standard Cauchy singular integral equation. By using the method described in [6], the algebraic equations corresponding to Eqs.15 and 16 are obtained and can be solved numerically. The dynamic stress intensity factors (DSIFs) can be defined and deduced as

$$K_{\mathrm{III}b_k}{}^{*}(p)=-\sqrt{c_k\pi}\left[\gamma_{11}R_k(1,p)\right]\qquad k=1,2,\cdots,n\qquad(17)$$
$$K_{\mathrm{III}a_k}{}^{*}(p)=\sqrt{c_k\pi}\left[\gamma_{11}R_k(-1,p)\right]\qquad k=1,2,\cdots,n\qquad(18)$$

where $R_k(\eta,p)=\sqrt{1-\eta^2}F_k(\eta,p)$.

The DSIFs in the time domain can be obtained by applying the numerical method developed by Miller and Guy[7]. Correspondingly, the dynamic energy release rates (DERRs) can be deduced as

$$G_{b_k} = 0.25 K_{\mathrm{III}b_k}{}^2(t) / \gamma_{11} \qquad k = 1,2,\cdots,n \tag{19}$$

$$G_{a_k} = 0.25 K_{\mathrm{III}a_k}{}^2(t) / \gamma_{11} \qquad k = 1,2,\cdots,n \tag{20}$$

Obviously, the imposed electric displacement impact doesn't contribute to the DSIFs and DERRs. And the electric displacements on crack surfaces $D_{y(j)}(x,0,t)$ consist of two parts. The first is the imposed $-D_0(x)\Psi_D(t)$. The second is the one caused by $-\tau_0(x)\Psi_\sigma(t)$, which is omitted here and can be easily obtained by using the solution of Eqs.15 and 16.

4. NUMERICAL EXAMPLE AND CONCLUSIONS

As an example, in this section we examine the DERRs of two cracks in the case of $-\tau_0(x)\Psi(t) \equiv -\tau_0 \mathrm{H}(t)$, and $a_1 = -b$, $b_1 = -a$, $a_2 = a$, $b_2 = b$. Without any loss in generality, in all our numerical procedure, we take $\tau_0 = 4.2 \times 10^6$ N/m^2. The DERRs are normalized by $G_0 = \pi c \varepsilon_{11(1)} \tau_0{}^2 / \left(2c_{44(1)}\varepsilon_{11(1)} + 2e_{15(1)}{}^2\right)$, and the normalized time is defined as $c_{T(1)} t / c$ with $c = \dfrac{b-a}{2}$. The upper material is taken as PZT-4 material constants of which are $c_{44} = 2.56 \times 10^{10}\,\mathrm{N/m^2}$, $e_{15} = 12.7\mathrm{C/m^2}$, $\varepsilon_{11} = 64.63 \times 10^{-10}\,\mathrm{C/Vm}$ and $\rho = 7.5 \times 10^3\,\mathrm{kg/m^3}$. As the effects of relative crack position on DERRs are investigated, the lower material is taken as $BaTiO_3$ material constants of which are $c_{44} = 4.4 \times 10^{10}\,\mathrm{N/m^2}$, $e_{15} = 11.4\mathrm{C/m^2}$,

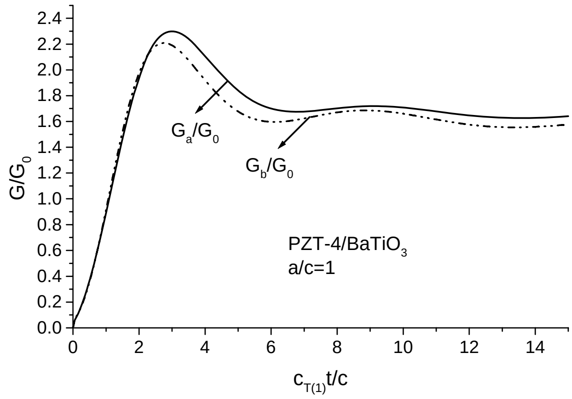

Fig.2. Normalized DERRs with normalized time at *x*=*a* and *x*=*b*

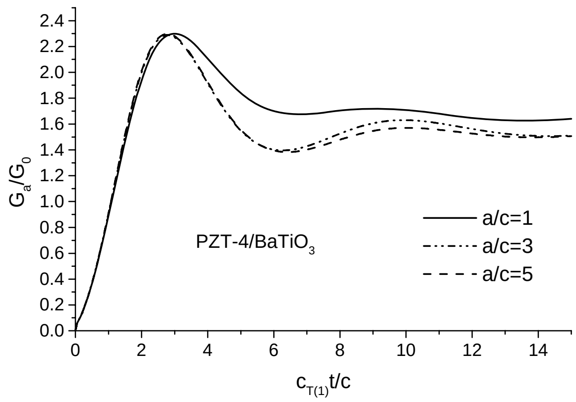

Fig.3. Normalized DERRs with normalized time for different relative crack positions

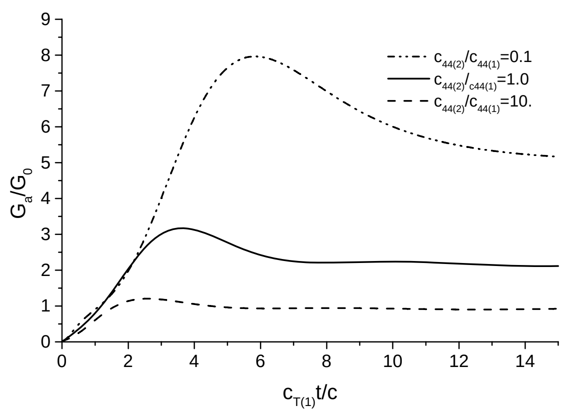

Fig.4. Normalized DERRs with normalized time for different ratios of elastic constants

$\varepsilon_{11} = 128.3\times10^{-10}$ C/Vm and $\rho = 5.7\times10^{3}$ kg/m^{3}. As the effects of material property parameters are investigated, except for the variation of the material constant pointed out in the corresponding figures, all the other material constants of the lower material are respectively set to be equal to that of the upper material, i.e. PZT-4.

Fig. 2 indicates that the DERRs at $x = a$ are higher than that at $x = b$. Therefore, under electromechanical impact, the cracks tend to propagate at first from inner crack tips for the case of two cracks. From Fig. 3, it is easily to know that the DERRs decrease slightly with a/c increasing. That is, the larger the distance between the two cracks, the weaker the oscillation is. Besides, the relative crack separation has little effects on the normalized time of reaching corresponding static value. As shown in Figs. 4-6, the peak values of DERRs generally decrease with the increasing of $c_{44(2)}/c_{44(1)}$ and/or $\varepsilon_{11(2)}/\varepsilon_{11(1)}$. However, the similar phenomena cannot be found for piezoelectric constants.

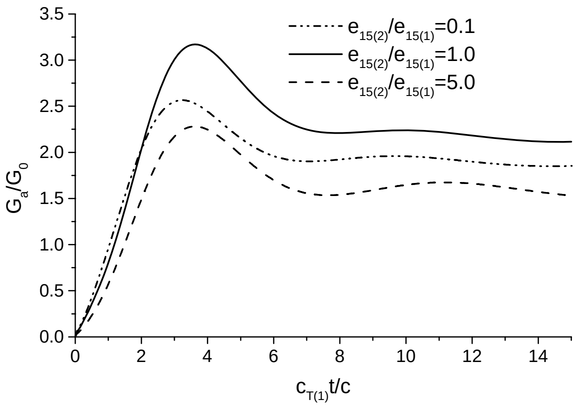

Fig.5. Normalized DERRs with normalized time for different ratios of piezoelectric constants

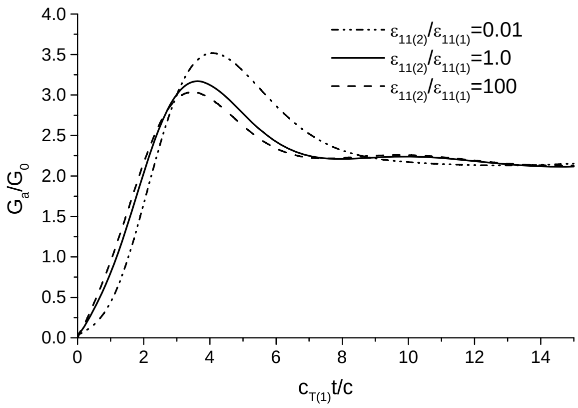

Fig.6. Normalized DERRs with normalized time for different ratios of dielectric constants

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