A new mass sensor based on thickness-twist edge modes in a piezoelectric plate

2007 Europhys. Lett. 77 28003

(http://iopscience.iop.org/0295-5075/77/2/28003)

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

January 2007

EPL, **77** (2007) 28003
www.epljournal.org
doi: 10.1209/0295-5075/77/28003

A new mass sensor based on thickness-twist edge modes in a piezoelectric plate

J. S. YANG[1,2] and A. K. SOH[3]

[1] *Key Laboratory for Advanced Materials and Rheological Properties of Ministry of Education, Xiangtan University Xiangtan, Huanan 411105, PRC*
[2] *Department of Engineering Mechanics, University of Nebraska - Lincoln, NE 68588, USA*
[3] *Department of Mechanical Engineering, The University of Hong Kong - Hong Kong*

received 10 August 2006; accepted in final form 22 November 2006
published online 19 January 2007

PACS 85.50.-n – Dielectric, ferroelectric, and piezoelectric devices

Abstract – We propose a new mass sensor based on thickness-twist edge modes in a piezoelectric plate of 6 mm crystals. By performing a theoretical analysis, a simple expression of sensitivity is obtained. The proposed sensor has an important advantage in the sense that it can be mounted away from the edge of the plate where the motion is insignificant and, thus, the operation of the device is unaffected.

Edge modes in plates and wedges are often used for resonators and acoustic wave sensors [1–5]. Compared to resonators based on bulk acoustic waves operating with infinite plate thickness-shear and thickness-stretch modes which have edge effects due to the finite sizes of real devices, edge modes are exact at the edge. An important advantage of edge modes is that away from the edge there is little motion, where mounting of the device can be achieved without affecting the device performance. The traditionally known edge modes are plate flexural [1,2] and thickness-stretch modes [3]. Recently edge thickness-twist modes in a piezoelectric plate of 6 mm crystals have been shown to exist [6]. The availability of these new edge modes suggests the possibility of producing new acoustic wave sensors with advantages. The objective of the present study is to show that new mass sensors can be made based on thickness-twist edge modes. In the study of acoustic wave mass sensors, researchers have been exploring various structures and modes to make different sensors. Some recent examples are the torsional and shear modes of shells [7,8].

Thickness-twist vibration modes of crystal plates are often used as the operating modes for resonators [9,10]. In addition to quartz plates which have been used for a long time, recently AlN and ZnO plates are of growing interest [11,12]. They are crystals of 6 mm symmetry. Plates with normal and in-plane six-fold axes are both being developed. Polarized ceramics like PZT are transversely isotropic and the material matrices for their linear behavior have the same structures as those of 6 mm crystals. Our analysis below is valid for both 6 mm crystals and polarized ceramics.

Consider the semi-infinite piezoelectric plate in fig. 1. The six-fold axis (or the poling direction of ceramics) is along x_3. The two major surfaces at $x_2 = \pm h$ are traction-free and are unelectroded. Thickness-twist motions of the plate are governed by

$$\begin{aligned} u_1 &= u_2 = 0, \\ u_3 &= u(x_1, x_2, t), \quad \phi = \phi(x_1, x_2, t), \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the displacement vector and ϕ is the electric potential. A function ψ can be introduced through [10,13]

$$\phi = \psi + \frac{e}{\varepsilon} u, \tag{2}$$

where $e = e_{15}$ and $\varepsilon = \varepsilon_{11}$ are the relevant piezoelectric and dielectric constants. The governing equations for u and ψ are [10,13]

$$\begin{aligned} \bar{c}\nabla^2 u &= \rho \ddot{u}, \\ \nabla^2 \psi &= 0, \end{aligned} \tag{3}$$

where ∇^2 is the two-dimensional Laplacian, ρ is the mass density, $\bar{c} = c + e^2/\varepsilon$, and $c = c_{44}$ is the relevant elastic constant. The nonzero stress and electric displacement components are [10,13]

$$\begin{aligned} T_{23} &= \bar{c} u_{,2} + e\psi_{,2}, \quad T_{31} = \bar{c} u_{,1} + e\psi_{,1}, \\ D_1 &= -\varepsilon \psi_{,1}, \quad D_2 = -\varepsilon \psi_{,2}, \end{aligned} \tag{4}$$

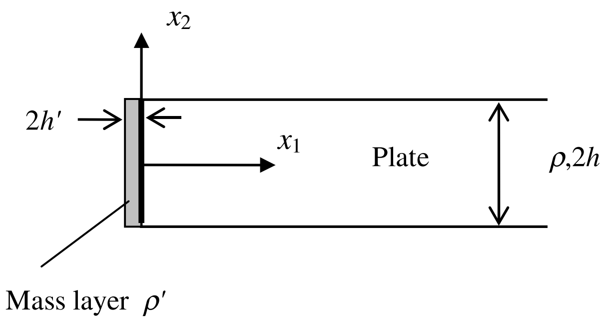

Fig. 1: A semi-infinite piezoelectric plate with an end electrode and an end mass layer.

where an index after a comma denotes partial differentiation with respect to the coordinate associated with the index. At the plate major surfaces, for traction-free and unelectroded boundary conditions we have

$$\begin{aligned} &T_{23}=0, \quad x_2=\pm h, \\ &D_2=0, \quad x_2=\pm h, \end{aligned} \tag{5}$$

or, equivalently, in terms of u and ψ,

$$u_{,2}=0, \quad \psi_{,2}=0, \quad x_2=\pm h. \tag{6}$$

At the left edge at $x_1=0$, we consider a thin mass layer of density ρ' and thickness $2h'$, as shown by the shaded area in fig. 1. Between the mass layer and the plate there is a very thin and grounded electrode with negligible mass, as shown by the thick line in fig. 1. The electrode merely provides a constraint on the electric potential. Mechanically the shear stress at the left edge carries the mass layer and is responsible for the acceleration of the mass layer according to Newton's law. Therefore, at the left edge, we have [14]

$$\begin{aligned} &T_{13}=\rho' h' \ddot{u}, \quad x_1=0, \\ &\phi=0, \quad x_1=0. \end{aligned} \tag{7}$$

For the right end of the plate at infinity, we require decaying behavior for edge modes to occur.

It can be verified by separation of variables or direct substitution that thickness-twist modes satisfying eqs. (3) and (6) can be classified into waves symmetric or antisymmetric in x_2, and they are given by [6]

$$\begin{aligned} &u=\cos\xi_2 x_2 A\exp(-\xi_1 x_1)\exp(i\omega t), \\ &\psi=\cos\xi_2 x_2 B\exp(-\xi_2 x_1)\exp(i\omega t), \\ &\xi_2=\frac{m\pi}{2h}, \quad m=0,2,4,\cdots, \\ &\text{and} \\ &u_3=\sin\xi_2 x_2 A\exp(-\xi_1 x_1)\exp(i\omega t), \\ &\psi=\sin\xi_2 x_2 B\exp(-\xi_2 x_1)\exp(i\omega t), \\ &\xi_2=\frac{m\pi}{2h}, \quad m=1,3,5,\cdots, \end{aligned} \tag{8}$$

respectively, where

$$\begin{aligned} \xi_1 &= \sqrt{\xi_2^2-\frac{\rho\,\omega^2}{\bar{c}}}=\sqrt{\frac{\rho}{\bar{c}}}\sqrt{\left(\frac{m\pi}{2h}\right)^2-\omega^2\frac{\bar{c}}{\rho}} \\ &= \frac{1}{v_T}\sqrt{\omega_m^2-\omega^2}, \\ v_T &= \sqrt{\frac{\bar{c}}{\rho}}, \quad \omega_m^2=\left(\frac{m\pi}{2h}\right)^2\frac{\bar{c}}{\rho}. \end{aligned} \tag{9}$$

A and B are undetermined constants. In particular, $m=0$ is called a face-shear mode. We will not consider this mode because from eq. (9) this mode cannot be an edge mode. The modes decay exponentially from the free edge at $x_1=0$ and therefore are called edge modes.

Equation (8) still needs to satisfy the boundary conditions on the minor surface at $x_1=0$. For time-harmonic motions, the mechanical boundary condition in eq. (7) takes the following form

$$T_{13}=-\omega^2\rho' h' u. \tag{10}$$

Substituting the modes in eq. (8) into eq. (10) and the electrical boundary condition at $x_1=0$ in eq. (7), we obtain

$$\begin{aligned} &\bar{c}(-\xi_1)A+e(-\xi_2)B=-\omega^2\rho' 2h' A, \\ &\frac{e}{\varepsilon}A+B=0. \end{aligned} \tag{11}$$

For nontrivial solutions we must require that

$$\begin{vmatrix} \bar{c}\xi_1-\omega^2\rho' 2h' & e\xi_2 \\ e/\varepsilon & 1 \end{vmatrix} = \bar{c}\xi_1-\omega^2\rho' 2h'-\frac{e^2}{\varepsilon}\xi_2=0, \tag{12}$$

which can be written as

$$\frac{1}{v_T}\sqrt{\omega_m^2-\omega^2}=\bar{k}_{15}^2\xi_2+\frac{\omega^2\rho' 2h'}{\bar{c}}, \tag{13}$$

where

$$\bar{k}_{15}^2=\frac{e^2}{\bar{c}\varepsilon} \tag{14}$$

is a piezoelectric coupling coefficient. Equation (13) determines the frequencies of the edge modes. For small ρ' and h', the root of eq. (13) is approximately given by

$$\begin{aligned} &\omega^2\cong v_T^2\xi_2^2(1-\bar{k}_{15}^4)\left(1-\bar{k}_{15}^2 m\pi\frac{\rho' 2h'}{\rho h}\right), \\ &m=1,2,3,\cdots. \end{aligned} \tag{15}$$

Equation (15) is true when the second term in the parentheses of the extreme right-hand side is much smaller than one. Therefore, eq. (15) is not valid for higher-order modes with a large m. This is fine because in applications usually lower-order modes with a small m is used. Results for a large m can be exactly determined from eq. (13) if necessary. If we denote the unperturbed frequencies when the mass layer is not present by

$$\hat{\omega}^2=\omega_m^2-v_T^2\bar{k}_{15}^4\xi_2^2=v_T^2\xi_2^2(1-\bar{k}_{15}^4)=(1-\bar{k}_{15}^4)\frac{\bar{c}}{\rho}\left(\frac{m\pi}{2h}\right)^2, \tag{16}$$

from eq. (15) we can obtain the frequency shift as

$$\frac{\omega - \hat{\omega}}{\hat{\omega}} \cong -\bar{k}_{15}^2 m\pi \frac{\rho' h'}{\rho h}. \tag{17}$$

We make the following observations from eq. (17):

i) The frequency shift is linear in the layer mass and is ideal for mass sensing;
ii) The inertial effect of the mass layer lowers the frequencies as expected;
iii) Higher-order modes imply higher sensitivity (this is true within a certain range of m);
iv) The mass-frequency effect shown in eq. (15) is proportional to $\bar{k}_{15}^2$ and, therefore, can only be detected in a piezoelectric plate, but not in an elastic plate. Higher piezoelectric coupling implies higher sensitivity.

In summary, we have obtained the mass sensitivity of certain exact edge modes in a piezoelectric plate of 6 mm crystals. The results suggest new mass sensors with advantages.

* * *

Support from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. HKU 7195/04E) is acknowledged.

REFERENCES

[1] Thurston R. N. and McKenna J., *IEEE Trans. Sonics Ultrasonics*, **21** (1974) 296.
[2] McKenna J., Boyd G. D. and Thurston R. N., *IEEE Trans. Sonics Ultrasonics*, **21** (1974) 178.
[3] Vangheluwe D. C. L. and Fletcher E. D., *Proceedings of the 35th Frequency Control Symposium*, Vol. **157** (1981).
[4] Kramer S. M., McBridge S. L., Mair H. D. and Hutchins D. A., *IEEE. Trans. Ultrasonics Ferroelectrics, Frequency Control*, **35** (1988) 253.
[5] Chamuel J. R., *Proceedings of the IEEE Ultrasonics Symp.*, Vol. **313** (1993).
[6] Yang J. S. and Guo S. H., *Appl. Phys. Lett.*, **88** (2006) 153506.
[7] Yang J. S. and Guo S. H., *Appl. Phys. Lett.*, **87** (2005) 214108.
[8] Yang J. S. and Guo S. H., *IEEE Trans. Ultrasonics, Ferroelectrics, Frequency Control*, **53** (2006) 1237.
[9] Mindlin R. D., *Int. J. Solids Structures*, **1** (1965) 141.
[10] Bleustein J. L., *J. Acoust. Soc. Am.*, **45** (1969) 614.
[11] Pang W., Zhang H. and Kim E. S., *IEEE Trans. Ultrasonics, Ferroelectrics, Frequency Control*, **52** (2005) 1239.
[12] Link M., Schreiter M., Weber J., Primig R., Pitzer D. and Cabl R., *IEEE Trans. Ultrasonics, Ferroelectrics, Frequency Control*, **53** (2006) 492.
[13] Yang J. S., *An Introduction to the Theory of Piezoelectricity* (Springer, New York) 2005.
[14] Yang J. S., *IEEE Trans. Ultrasonics, Ferroelectrics, Frequency Control*, **51** (2004) 1199.