Efficient Linear Macromodeling via Least-Squares Response Approximation

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Abstract— We present a least-squares (LS) algorithm for rational function macromodeling of port-to-port responses with discrete-time sampled data. The core routine involves over-determined equations and filtering operation, and avoids numerical-sensitive calculation and initial pole assignment. We demonstrate the fast computation and excellent accuracy and robustness, even with noisy data, in stable response approximation.

I. INTRODUCTION

In deep-submicron VLSI design, the signal integrity analysis constantly requires efficient modeling and high-frequency simulation of passive structures such as packages and interconnect networks [1], [2]. A full-wave electromagnetic (EM) analysis over a global system is impractical. Therefore, model reduction techniques and rational function approximation algorithms such as Vector Fitting (VF) [3] have been used to construct a reduced system for efficient simulation. However, frequency-domain macromodeling requires a spectral conversion using FFT in the VF-based signal integrity analysis, which requires complicated measurement and relatively longer data sequences to be captured. Time-domain vector fitting (TD-VF) [4], [5] is developed for time-domain impulse response approximation, which can approximate truncated response effectively. According to the discrete-time nature of time-sampled response, discrete-time domain vector fitting (TD-VFz) [6] has been recently proposed for more accurate approximation.

On the other hand, in digital filter design, [7] proposes an efficient IIR approximation method of FIR filters. The idea is to design an allpass filter with certain properties, and separate numerator and denominator calculation. It involves over-determined equations and digital filtering operations without state-space realization, eigenvalue computation or initial pole assignment. Therefore, the algorithm avoids numerical-sensitive calculation and is superior to other IIR approximation algorithms, in terms of both accuracy and speed [8].

In this paper, we extend the least-squares (LS) IIR approximation idea in [7] to interconnect macromodeling. The proposed algorithm respects the discrete nature of input/output sequences and works exclusively in the discrete-time domain, and is particularly suitable for generating macromodels based on truncated transient response analysis. Moreover, the algorithm enjoys simple coding and is numerically robust. Numerical examples then confirm the remarkable efficiency and accuracy of the algorithm.

II. ALGORITHM

A. Introduction

Time-domain identification techniques fit the rational function

$$\widehat{f}(s) = \frac{P(s)}{Q(s)} = \sum_{u=0}^{M} \frac{p(u) s^u}{q(u) s^u}, \tag{1}$$

where $p(u), q(u) \in \Re, q(0) = 1$, to the desired response $f(s)$ at a set of discrete-time calculated/sampled input and output data points $G[k]$ and $H[k]$, where $k = 1, 2, \cdots, L$. Usually the rational function approximation is solved by non-linear optimization or iterative calculation, and needs numerical-sensitive and initial-guess-sensitive calculation. According to Walsh's theorem [9], the numerator coefficients calculation becomes an interpolation problem if the denominator (poles) is known. Therefore, the approximation problem can be separated into two parts: denominator (poles) calculation and numerator (zeros) calculation. In the proposed algorithm, the obtained output response is first deconvoluted with the input pulse response such that $G[0] = 1$ and $G[k] = 0$ for $k = 1, 2, \cdots, L$, in order to fit the algorithm.

B. Denominator calculation

The denominator can be calculated by designing an allpass filer which redistributes the output energy of the filter in the time domain and thus reduces the approximation error [7]. The algorithm runs iteratively, which involves a digital filtering operation (convolution) and a set of over-determined equation solutions to minimize (2),

$$F^{(k)} = \left(\mathbf{B}^{(k)} \mathbf{q}^{(k)} - \mathbf{d}^{(k)} \right)^T \left(\mathbf{B}^{(k)} \mathbf{q}^{(k)} - \mathbf{d}^{(k)} \right), \tag{2}$$

where $\mathbf{q}^{(k)} = \begin{bmatrix} q^{(k)}(M) & \cdots & q^{(k)}(1) \end{bmatrix}^T$, $\mathbf{d}^{(k)} = -\begin{bmatrix} 0 & \cdots & 0 & x^{(k)}(0) & \cdots & x^{(k)}(L-M-1) \end{bmatrix}^T$,

$$\mathbf{B}^{(k)} = \begin{bmatrix} x^{(k)}(0) & 0 & \cdots & 0 \\ x^{(k)}(1) & x^{(k)}(0) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ x^{(k)}(M-1) & x^{(k)}(M-2) & \cdots & x^{(k)}(0) \\ \vdots & \vdots & & \vdots \\ x^{(k)}(L-1) & x^{(k)}(L-2) & \cdots & x^{(k)}(L-M) \end{bmatrix},$$

$$Q^{(0)}(z) = 1, \quad Q^{(k)}(z) = 1 + \sum_{n=1}^{M} q^{(k)}(n) z^{-n},$$

$$X^{(k)}(z) = \frac{z^{-L}H(z^{-1})}{Q^{(k-1)}(z)} = \sum_{n=0}^{\infty} x^{(k)}(n) z^{-n}, \quad H(z) \text{ is}$$

the deconvoluted impulse response, and k is the number of the iterations.

The algorithm converges after sufficient iterations [7], [10], and we take $Q(z) := Q^{(k)}(z)$. It is proved that for arbitrary $X^{(k)}(z)$ which minimizes (2), the roots of the denominator are strictly inside the unit circle in the z-domain [7], and therefore the approximant is always stable. Compared to other algorithms, the proposed algorithm does not require eigenvalue calculation and initial-pole assignment, which significantly affects the accuracy and convergence of the iterative algorithm. Furthermore, the approximant phase response will not be affected by any unstable pole flipping technique involved in other iterative algorithms such as VF.

C. Numerator calculation

Suppose the denominator function (poles) converges, the final step is to reconstruct the rational function (1). By the orthogonality principle, the numerator $P(z)$ is calculated by the following relation,

$$\left\langle H(z) - \frac{P(z)}{Q(z)}, \frac{z^k}{Q(z)} \right\rangle = 0, \tag{3}$$

where $k = 0, 1, \ldots, M$.

Walsh's Theorem [9]: Among the set of rational functions (1), with prescribed poles $\alpha_1, \alpha_2, \ldots, \alpha_n$ that are fixed and located in $|z| < 1$, the best approximation in the LS sense to $H(z)$ (analytic in $|z| > 1$ and continuous in $|z| \geq 1$) is the unique function that interpolates to $H(z)$ in all the points. $z = \infty, 1/\alpha_1^*, 1/\alpha_2^*, \ldots, 1/\alpha_n^*$, where * denotes complex conjugate.

By the Walsh's Theorem, (3) becomes an interpolation problem of $H(z)$ to $\frac{P(z)}{Q(z)}$. Since the approximant is an impulse response approximation of a system, the interpolation problem can be described as an input-output description of a digital filter operation in Fig. 1 and calculated by the following relationship,

$$P(z) = H(z)Q(z) - z^{-(M+1)}Q(z^{-1})R(z), \tag{4}$$

where $R(z) = z^{-L}U(z^{-1})$, $U(z) = \left(z^{-L}H(z^{-1})\right) \otimes A(z)$ and $A(z) = z^{-M}Q(z^{-1})/Q(z)$, with $\otimes$ denoting convolution. A digital-time macromodel can be obtained from $\frac{P(z)}{Q(z)}$. The system can then be used in the simulation environment directly or transformed into a continuous-time system using bilinear (Tustin) transformation or other conformal mapping techniques.

III. REMARKS

Several properties of the proposed algorithm are in order:

1) Correct macromodel order selection can facilitate the simulation process without significant response loss.
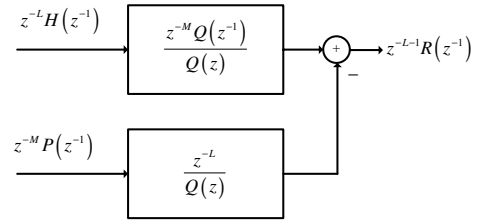

Fig. 1. Filtering operation in (4).

A heuristic method is proposed for order selection recently [11]. In [7], the Hankel singular value plot (HSVP) is introduced as an efficient means to determine the number of approximant order. An HSVP is a graphical representation of the singular values of the Hankel matrix of $H(z)$ (5) that consists of the coefficients of the impulse response. In practical cases, the graph shows the similarity between the magnitude of singular values and approximation error. Therefore, it gives a quantitative analysis for order selection.

$$H = \begin{bmatrix} h_1 & h_2 & \cdots & h_L \\ h_2 & h_3 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h_L & 0 & \cdots & 0 \end{bmatrix}. \tag{5}$$

2) The convergence of VF will be seriously affected by even a small spectral noise (disturbance), e.g, when SNR=30dB [12], which always happens in practical measurement. In the proposed algorithm, the noise in the measured response signal is automatically suppressed during the digital filter operation. And the noise only affects the algorithmic convergence slightly.

3) RF circuit may require a multi-pole macromodeling for its resonant response approximation. Sometimes VF may be inaccurate due to its partial fraction basis limitation. Recently, VF solves the problem by modifying the basis and its state-space realization [13]. However, the modification increases the computational complexity and requires manual order-multiplicity selection. Since the proposed algorithm directly calculates the denominator and numerator, it can identify multi-pole system automatically without increasing the computational complexity.

4) Transmission-line structure contains time delay factor (the propagation of the main pulse), for which the frequency-domain macromodeling can apply an delay extraction post-processing step to reduce approximant order significantly [14]. However, this post-processing step cannot be calculated directly. In the proposed algorithm, the time-sampled signal can be shifted directly in calculation to extract the time delay factor easily.

IV. NUMERICAL EXAMPLES

The proposed algorithm is coded in Matlab m-script (text) files and run in the Matlab 7.4 environment on a 1GB-RAM
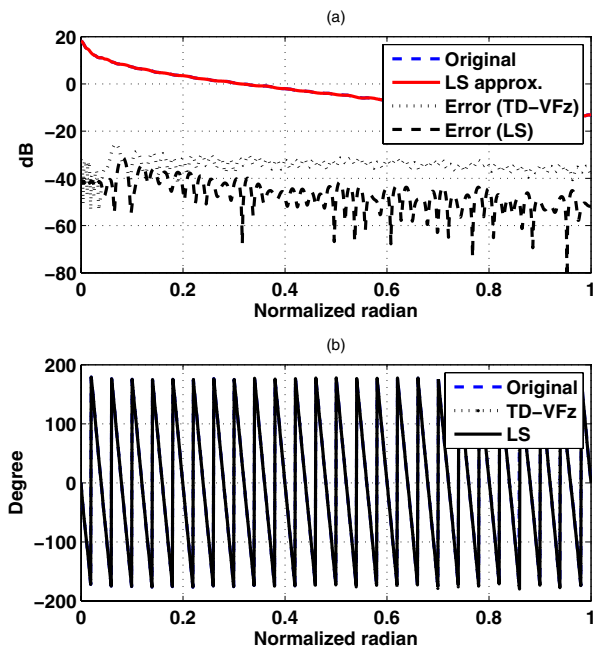
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Fig. 2. (a) Magnitude responses and (b) phase responses of the numerical example in digital frequency.
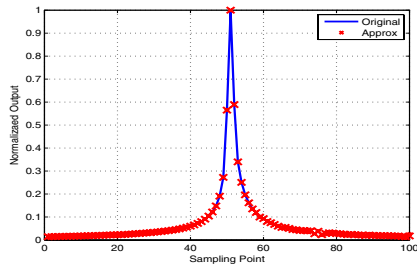

Fig. 3. Normalized time-domain response of the numerical example.

3.4GHz PC. Examples are used to show the efficiency and accuracy of the proposed algorithm. The numerical example arises from modeling a differential transmission channel on a full mesh ATCA backplane [15]. The time-domain response is computed from its frequency response ranging from 50 MHz to 15GHz. The signal is normalized and fitted using the proposed algorithm with a 62-pole approximant. Time samples are taken at 0.67ps intervals for the first 425 points. The algorithm requires 21 iterations (0.52 seconds) to converge. Fig. 2 and Fig. 3 plot the normalized digital frequency-domain responses and the normalized time-domain responses of the converged approximant, demonstrating the excellent fitting accuracy in both time and frequency domains. The model is also modeled using TD-VFz [6] and modified VF (MVF) [15], where MVF considers time delay estimation to reduce approximant size. The quantitative data are shown in Table I. It shows that the proposed algorithm gives a more accurate macromodel with a smaller model size. It also significantly

TABLE I
COMPARISON BETWEEN THE PROPOSED ALGORITHM AND OTHER ALGORITHMS.

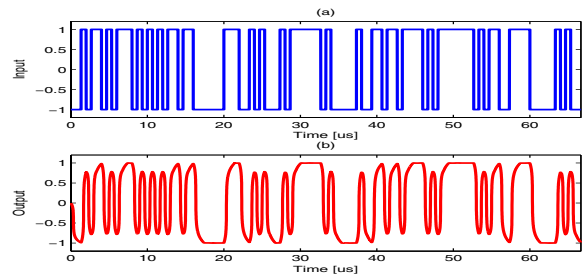|	LS	TD-VFz [6]	MVF [15]	LS with noise
Number of poles	62	62	66	62
Relative error (dB)	-47	-37	-38	-31
CPU Time (sec.)	0.5	8	33	0.5

Fig. 4. Time-domain output response of the macromodel with a random high-frequency input signal.

reduces the approximation time. This illustrates the power of the proposed algorithm in achieving accurate approximation in a short computation time. The rational macromodels can then be easily integrated into simulation environment. For example, an output signal response of a high-frequency random input signal to the backplane example is shown in Fig. 4 using Matlab. Furthermore, the digital-time model is converted into continuous-time model via bilinear transformation, which is shown in Fig. 5.

Properties of the algorithm are investigated. Fig. 6 shows the relative error in each iteration. It shows that the algorithm converges quickly, which is similar to the linear-phase filter approximation in [7]. In general, the algorithm converges within 15 to 25 iterations. Also, we investigate the use of the HSV in guiding the model order selection. Fig. 7 shows the HSVs found by the impulse response of the backplane system, as well as the relative error of different approximant
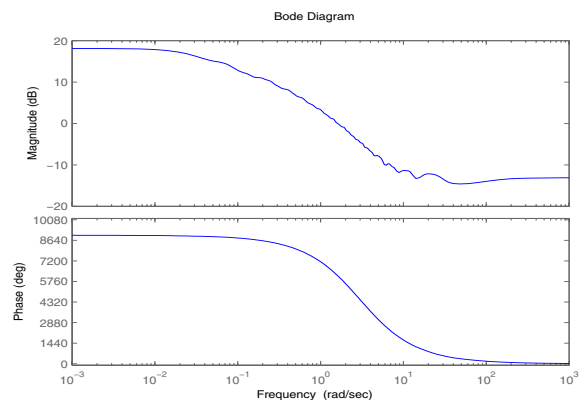

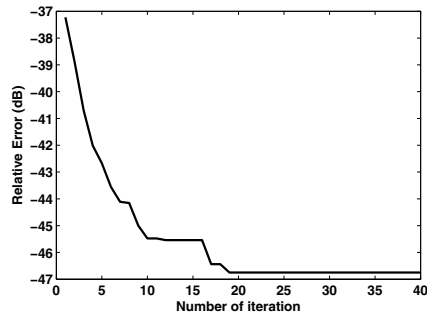Fig. 5. Bode plot of the numerical example in continuous frequency.

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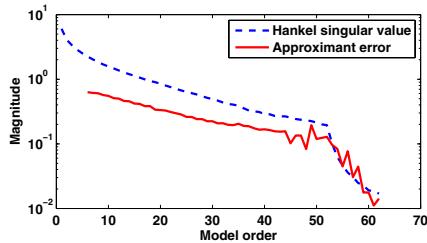Fig. 6. The relative error curve of the numerical example with respect to the number of iterations.

Fig. 7. Hankel singular value of the impulse system and the approximant error in the numerical example.
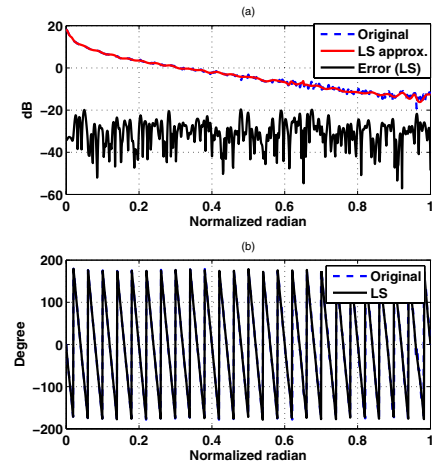

Fig. 8. (a) Magnitude responses and (b) phase responses of the numerical example with SNR=-30dB in digital frequency.

orders. An evidential correlation can be seen between these two parameters, with similar slope and tipping point (i.e., 52th order in this example). We can therefore choose the approximant order to be bigger than or equal to the corner cut-off point. Finally, we also study the robustness of the proposed algorithm under noisy input empirically. We repeat example one with the same approximant specification but with the output sequences corrupted by white noise under a signal-to-noise ratio (SNR) of 30dB. Fig. 8 plots the normalized digital frequency-domain response. In this case, the proposed algorithm converges in 0.48 CPU seconds, with an -31 dB error, thereby demonstrating that the proposed algorithm is robust against noisy data.

V. CONCLUSION

This paper has extended the LS IIR approximation technique to the linear interconnect macromodeling algorithm of port-to-port responses in time-sampled data. The proposed algorithm avoids the numerical-sensitive calculation and initial pole assignment. Algorithmic convergence and criterion for model order selection have been elaborated. Application examples have confirmed that the proposed algorithm exhibits efficient computation and produces highly accurate approximants, in terms of both time and frequency responses, when compared to conventional algorithms.

ACKNOWLEDGMENT

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