ROBUST JOINT LOCALIZATION AND TIME SYNCHRONIZATION IN WIRELESS SENSOR NETWORKS WITH BOUNDED ANCHOR UNCERTAINTIES

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ABSTRACT

A unified framework to jointly solve the problems of localization and synchronization at the same time is presented in this paper. The joint approach is attractive because it can solve both localization and synchronization using the same set of message exchanges, which is extremely important for energy saving in wireless sensor networks. The inaccuracy of anchor locations and timings is taken into account to provide accurate joint localization and synchronization. The anchor uncertainties are assumed to be bounded, but knowledge of the statistics of anchor uncertainties is not required. The problem is formulated into a linear model with uncertainties on both sides of the equation. A robust joint estimator is then proposed based on minimizing the worst-case mean square error and the solution is obtained by solving a semidefinite programming problem. Simulation results show that the proposed estimator outperforms the traditional least squares estimator at the cost of higher computational complexity.

Index Terms— Localization, time synchronization, anchor uncertainty, semidefinite programming

1. INTRODUCTION

Because of the wide applications of wireless sensor networks (WSNs) in environmental monitoring, natural disaster prediction, health care, manufacturing and transportation, WSNs have attracted enormous interest in recent years. In WSNs, localization is the basis of applications which require accurate locations of sensor nodes, such as environment monitoring, emergency rescue and geographic routing. On the other hand, synchronization supports functions such as time-based channel sharing, power scheduling, and time-based localization [1].

While localization is traditionally studied from the signal processing point of view [2], time synchronization is mainly studied from protocol design of view [3]. As a result, these two problems have been investigated separately for a long time. However, localization and time synchronization have very close relationships and share many aspects in common.

For time-based (i.e., time-of-arrival) localization algorithms, time synchronization is even a prerequisite [2].

Based on the close relationships between localization and time synchronization, it is natural to explore the possibility of formulating them into a unified framework and solve the two problems at the same time. The joint localization and synchronization approach is extremely attractive in WSNs because the joint approach makes it possible to carry out localization and time synchronization with only one set of data package exchanges, rather than two. This is extremely crucial for WSNs as the power and memory of the sensor nodes are very limited.

Recently, some pioneering research works noticed the similarities between the problem of localization and time synchronization [4]. However, [4] only explores the possibility of jointly implementing localization and time synchronization at the protocol level. In [5], it was the first time that a unified framework for joint localization and time synchronization was proposed from signal processing perspective, assuming accurate anchors.

In this paper, the results in [5] are extended to the case where there are uncertainties in anchor locations and timings. When a hierarchical method is used to localize and synchronize a large sensor network, some newly localized and synchronized sensors act as anchors to localize and synchronize other nodes. These new anchors are subject to uncertainties in their own locations and timings and the uncertainties need to be taken into account for error propagation relief. Since accurate statistics of the anchor uncertainties are usually unknown to the node, no assumption about the distribution of the anchor uncertainties is made in our study. We only assume that the anchor uncertainties are bounded.

2. SYSTEM MODEL

We consider a single node joint localization and time synchronization in a WSN, where only one node needs to be localized and synchronized to the anchors at a time. There are L ($L \geq 3$) anchors with known locations and timings. The lth anchor A_l is located at $\boldsymbol{a}_l^o = [a_{xl}^o, a_{yl}^o]^T$ with time

skew θ_{sl}^o and time offset θ_{0l}^o. A node B with unknown location $\boldsymbol{x} = [x, y]^T$, time skew θ_s and time offset θ_o uses the time-stamps in two-way message exchanges with anchors to estimate its location and timing parameters.

Assume there are M rounds of time-stamp exchanges between node B and anchor A_l. As shown in Figure 1, the mth message is sent from node B at time T_{lm} and is received by A_l at time R_{lm}. Then, anchor A_l replies node B with another message sent at time $\bar{T}_{lm}$ and is received by node B at time $\bar{R}_{lm}$. In the reply message from anchor A_l to node B, the time-stamps R_{lm} and $\bar{T}_{lm}$ at the anchor side are also included. Therefore, node B has all the time-stamp information $\{T_{lm}, R_{lm}, \bar{T}_{lm}, \bar{R}_{lm}\}$. Note that R_{lm} and $\bar{T}_{lm}$ are measured with respect to the clock of anchors, while T_{lm} and $\bar{R}_{lm}$ are measured with respect to the clock of node B. The exchanged time-stamps can be modeled as [6]

$$T_{lm} = \frac{\theta_s}{\theta_{sl}^o} R_{lm} - \theta_s(t_l + n_{lm}) + \theta_0 - \frac{\theta_s}{\theta_{sl}^o}\theta_{0l}^o, \quad (1)$$

$$\bar{R}_{lm} = \frac{\theta_s}{\theta_{sl}^o} \bar{T}_{lm} + \theta_s(t_l + \bar{n}_{lm}) + \theta_0 - \frac{\theta_s}{\theta_{sl}^o}\theta_{0l}^o, \quad (2)$$

where $t_l = \|\boldsymbol{x} - \boldsymbol{a}_l^o\|/c$ is the propagation delay between node B and anchor A_l, with c being the speed of light. Symbols n_{lm} and $\bar{n}_{lm}$ are the time-of-arrival (TOA) detection errors, which are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance σ_n^2 [6].

When there are uncertainties in the anchors, we can only have the observed (but not true) values of the location, time skew and time offset of anchors

$$\boldsymbol{a}_l = \boldsymbol{a}_l^o - \Delta\boldsymbol{a}_l, \ \theta_{sl} = \theta_{sl}^o - \Delta\theta_{sl}, \ \theta_{0l} = \theta_{0l}^o - \Delta\theta_{0l}, \quad (3)$$

where $\Delta\boldsymbol{a}_l = [\Delta a_{xl}, \Delta a_{yl}]^T$, $\Delta\theta_{sl}$ and $\Delta\theta_{0l}$ are the error in location, time skew and time offset of the lth anchor. To make the discussion more general, we only assume the anchor uncertainties are bounded, but make no assumption about the distribution of the anchor uncertainties.

3. PROBLEM FORMULATION

Dividing both sides of (1) and (2) by θ_s, and rearranging the equations, we have

$$-t_l = -\frac{R_{lm} - \theta_{0l}^o}{\theta_{sl}^o} + T_{lm}\theta_1 - \theta_2 + n_{lm}, \quad (4)$$

$$-t_l = \frac{\bar{T}_{lm} - \theta_{0l}^o}{\theta_{sl}^o} - \bar{R}_{lm}\theta_1 + \theta_2 + \bar{n}_{lm}, \quad (5)$$

where $\theta_1 \triangleq 1/\theta_s$ and $\theta_2 \triangleq \theta_0/\theta_s$ have been introduced. Squaring both sides of (4) and (5) and introducing

$$\boldsymbol{\xi} = [\theta_1^2/2, \ (\theta_2^2 - \|\boldsymbol{x}\|/c^2)/2, \ \theta_1\theta_2, \ \theta_1, \ \theta_2, \ \boldsymbol{x}^T]^T, \quad (6)$$

the equations in (4) and (5) can be formulated into a linear model as

$$\boldsymbol{A}^o\boldsymbol{\xi} = \boldsymbol{b}^o - \boldsymbol{e}, \quad (7)$$

where $\boldsymbol{A}^o \triangleq [\boldsymbol{A}_1^o, \boldsymbol{A}_2^o, \cdots, \boldsymbol{A}_L^o]^T$ and $\boldsymbol{b}^o \triangleq [\boldsymbol{b}_1^o, \boldsymbol{b}_2^o, \cdots, \boldsymbol{b}_L^o]^T$ with $\boldsymbol{A}_l^o$ and $\boldsymbol{b}_l^o$ shown in (8) at the top of next page. The error vector in (7) is defined as $\boldsymbol{e} \triangleq [e_{11}, \bar{e}_{11}, \cdots, e_{LM}, \bar{e}_{LM}]^T$ with

$$e_{lm} \approx 2(-T_{lm}\theta_1 + \theta_2 + (R_{lm} - \theta_{0l})/\theta_{sl})n_{lm},$$
$$\bar{e}_{lm} \approx 2(\bar{R}_{lm}\theta_1 - \theta_2 - (\bar{T}_{lm} - \theta_{0l})/\theta_{sl})\bar{n}_{lm}, \quad (9)$$

where the approximation in (9) is due to the fact that θ_{sl}^o and θ_{0l}^o are approximated by their observations θ_{sl} and θ_{0l}, and the second order terms of the anchor uncertainties and TOA detection errors have been ignored in the operations that lead to $\boldsymbol{e}$.

Substituting $\boldsymbol{a}_l^o = \boldsymbol{a}_l + \Delta\boldsymbol{a}_l$, $\theta_{sl}^o = \theta_{sl} + \Delta\theta_{sl}$, and $\theta_{0l}^o = \theta_{0l} + \Delta\theta_{0l}$ into (7) and using the first-order Taylor series approximation $1/(\theta_{sl} + \Delta\theta_{sl}) \approx 1/\theta_{sl} - \Delta\theta_{sl}/\theta_{sl}^2$, (7) can be re-written as

$$(\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{\xi} = \boldsymbol{b} + \Delta\boldsymbol{b} - \boldsymbol{e}, \quad (10)$$

where $\boldsymbol{A}$ and $\boldsymbol{b}$ are $\boldsymbol{A}^o$ and $\boldsymbol{b}^o$ with $\boldsymbol{a}^o$, θ_{sl}^o and θ_{0l}^o replaced by $\boldsymbol{a}$, θ_{sl} and θ_{0l}, respectively. Symbols $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{b}$ represent the perturbations, which are unobservable, in $\boldsymbol{A}$ and $\boldsymbol{b}$, respectively, and are given by

$$\Delta\boldsymbol{A} = [\frac{2}{c^2}[\Delta\boldsymbol{a}_1, \cdots, \Delta\boldsymbol{a}_L]^T \otimes \boldsymbol{1}_{2M\times1}, \boldsymbol{0}_{2LM\times5}],$$

$$\Delta\boldsymbol{b} = \frac{2}{c^2}[\boldsymbol{a}_1^T\Delta\boldsymbol{a}_1, \cdots, \boldsymbol{a}_L^T\Delta\boldsymbol{a}_L]^T \otimes \boldsymbol{1}_{2M\times1}, \quad (11)$$

where $\otimes$ is the Kronecker product.

4. ROBUST JOINT LOCALIZATION AND SYNCHRONIZATION

In the lack of information about the uncertainties $\Delta\boldsymbol{A}$ and $\Delta\boldsymbol{b}$, the traditional least squares (LS) method is usually used to solve the problem in (10). However, if the uncertainties are known to be bounded, further improvement on the performance is possible by taking the bounds into account. In this section, a robust minimax approach is presented to solve (10) with the information that $\Delta\boldsymbol{A}$, covariance matrix $\boldsymbol{C}_{\Delta\boldsymbol{b}} = E\{\Delta\boldsymbol{b}\Delta\boldsymbol{b}^T\}$ and $\boldsymbol{\xi}$ are bounded.

To solve the problem in (10) with bounded uncertainties, the robust minimax approach seeks a linear estimator $\hat{\boldsymbol{\xi}} = \boldsymbol{H}\boldsymbol{b}$, where matrix $\boldsymbol{H}$ is chosen to minimize the MSE:

$$\begin{aligned} MSE &= E\{\|\hat{\boldsymbol{\xi}} - \boldsymbol{\xi}\|^2\} \\ &= E\{\|\boldsymbol{H}\boldsymbol{b} - \boldsymbol{\xi}\|^2\} \\ &= E\{\|\boldsymbol{H}((\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{\xi} - (\Delta\boldsymbol{b} - \boldsymbol{e})) - \boldsymbol{\xi}\|^2\} \\ &= \boldsymbol{\xi}^T(\boldsymbol{I} - \boldsymbol{H}(\boldsymbol{A} + \Delta\boldsymbol{A}))^T(\boldsymbol{I} - \boldsymbol{H}(\boldsymbol{A} + \Delta\boldsymbol{A}))\boldsymbol{\xi} \\ &\quad + Tr(\boldsymbol{H}\boldsymbol{C}\boldsymbol{H}^T), \quad (12) \end{aligned}$$

$$
\boldsymbol{A}_l^o = 2 \begin{bmatrix} T_{l1}^2 & 1 & -T_{l1} & -T_{l1}(R_{l1}-\theta_{0l}^o)/\theta_{sl}^o & (R_{l1}-\theta_{0l}^o)/\theta_{sl}^o & \boldsymbol{a}_l^{oT}/c^2 \\ \bar{R}_{l1}^2 & 1 & -\bar{R}_{l1} & -\bar{R}_{l1}(\bar{T}_{l1}-\theta_{0l}^o)/\theta_{sl}^o & (\bar{T}_{l1}-\theta_{0l}^o)/\theta_{sl}^o & \boldsymbol{a}_l^{oT}/c^2 \\ & & & \vdots & & \\ T_{lM}^2 & 1 & -T_{lM} & -T_{lM}(R_{lM}-\theta_{0l}^o)/\theta_{sl}^o & (R_{lM}-\theta_{0l}^o)/\theta_{sl}^o & \boldsymbol{a}_l^{oT}/c^2 \\ \bar{R}_{lM}^2 & 1 & -\bar{R}_{lM} & -\bar{R}_{lM}(\bar{T}_{lM}-\theta_{0l}^o)/\theta_{sl}^o & (\bar{T}_{lM}-\theta_{0l}^o)/\theta_{sl}^o & \boldsymbol{a}_l^{oT}/c^2 \end{bmatrix}^T, \quad \boldsymbol{b}_l^o = \begin{bmatrix} \|\boldsymbol{a}_l^o\|^2/c^2 - (R_{l1}-\theta_{0l}^o)^2/\theta_{sl}^{o2} \\ \|\boldsymbol{a}_l^o\|^2/c^2 - (\bar{T}_{l1}-\theta_{0l}^o)^2/\theta_{sl}^{o2} \\ \vdots \\ \|\boldsymbol{a}_l^o\|^2/c^2 - (R_{lM}-\theta_{0l}^o)^2/\theta_{sl}^{o2} \\ \|\boldsymbol{a}_l^o\|^2/c^2 - (\bar{T}_{lM}-\theta_{0l}^o)^2/\theta_{sl}^{o2} \end{bmatrix}^T
$$
(8)

where $\boldsymbol{C} \triangleq \boldsymbol{C}_{\Delta b} + E\{\boldsymbol{e}\boldsymbol{e}^T\}$ is the covariance matrix of the error on the right side of (10), and $Tr(\cdot)$ is the trace of a matrix. In (12), in addition to the optimization variable $\boldsymbol{H}$, the values of $\boldsymbol{\xi}$, $\boldsymbol{C}$ and $\Delta\boldsymbol{A}$ are also unknown. Therefore, we minimize the worst-case MSE across all possible values of $\boldsymbol{\xi}$, $\boldsymbol{A}$ and $\boldsymbol{C}$:

$$
\min_{\boldsymbol{H}} \max_{\|\boldsymbol{\xi}\|\leq N, \boldsymbol{A}\in\mathcal{U}_A, \boldsymbol{C}\in\mathcal{U}_C} E\{\|\boldsymbol{H}\boldsymbol{b}-\boldsymbol{\xi}\|^2\}
$$
$$
= \min_{\boldsymbol{H}} \max_{\|\boldsymbol{\xi}\|\leq N, \boldsymbol{A}\in\mathcal{U}_A, \boldsymbol{C}\in\mathcal{U}_C} \{\boldsymbol{\xi}^T(\boldsymbol{I}-\boldsymbol{H}\boldsymbol{A})^T(\boldsymbol{I}-\boldsymbol{H}\boldsymbol{A})\boldsymbol{\xi}
$$
$$
+ Tr(\boldsymbol{H}\boldsymbol{C}\boldsymbol{H}^T)\} \quad (13)
$$

where N is a positive constant. The uncertainty sets of $\boldsymbol{A}$ and $\boldsymbol{C}$ are defined by

$$
\mathcal{U}_A = \{\boldsymbol{A}+\Delta\boldsymbol{A} : \Delta\boldsymbol{A} \in \mathbb{R}^{2LM\times 7}, \|\Delta\boldsymbol{A}\| \leq \rho_A\} \quad (14)
$$
$$
\mathcal{U}_C = \{\boldsymbol{C}_e + \boldsymbol{C}_{\Delta b} : \boldsymbol{C}_{\Delta b} \in \mathbb{R}^{2LM\times 2LM},
$$
$$
\|\boldsymbol{C}_{\Delta b}\| \leq \rho_C, \boldsymbol{C}_e + \boldsymbol{C}_{\Delta b} \succeq 0\} \quad (15)
$$

where ρ_A and ρ_C are nonnegative constants, $\boldsymbol{C}_e = 4\sigma_n^2 \boldsymbol{T}\boldsymbol{T}^T$ is the covariance of $\boldsymbol{e}$ with

$$
\boldsymbol{T} = \begin{bmatrix} -T_{lm}\theta_1 + \theta_2 + (R_{lm}-\theta_{0l})/\theta_{sl} \\ \bar{R}_{lm}\theta_1 - \theta_2 - (\bar{T}_{lm}-\theta_{0l}/\theta_{sl} \\ \vdots \\ -T_{LM}\theta_1 + \theta_2 + (R_{LM}-\theta_{0l})/\theta_{sL} \\ \bar{R}_{LM}\theta_1 - \theta_2 - (\bar{T}_{LM}-\theta_{0L})/\theta_{sL} \end{bmatrix}. \quad (16)
$$

The minimax problem in (13) can be formulated into the following semidefinite programming (SDP) formulation [7]

$$
\min_{\tau,\lambda,t_1,t_2,\boldsymbol{Y},\boldsymbol{H}} \{N^2\tau + t_1 + \rho_C t_2\}
$$
subject to
$$
\begin{bmatrix} (\tau-\lambda)\boldsymbol{I} & (\boldsymbol{I}-\boldsymbol{H}\boldsymbol{A})^T & 0 \\ \boldsymbol{I}-\boldsymbol{H}\boldsymbol{A} & \boldsymbol{I} & -\rho_A\boldsymbol{H} \\ 0 & -\rho_A\boldsymbol{H}^T & \lambda\boldsymbol{I} \end{bmatrix} \succeq 0
$$
$$
\begin{bmatrix} t_1 & \boldsymbol{h}^T \\ \boldsymbol{h} & \boldsymbol{I} \end{bmatrix} \succeq 0
$$
$$
\begin{bmatrix} \boldsymbol{Y} & \boldsymbol{H}^T \\ \boldsymbol{H} & \boldsymbol{I} \end{bmatrix} \succeq 0
$$
$$
\begin{bmatrix} t_2 & \boldsymbol{y}^T \\ \boldsymbol{y} & t_2\boldsymbol{I} \end{bmatrix} \succeq 0 \quad (17)
$$

where $\boldsymbol{y} = vec(\boldsymbol{Y})$ and $\boldsymbol{h} = vec(\boldsymbol{H}\boldsymbol{C}_e^{1/2})$, with $vec(\cdot)$ the vectorization operator and $\boldsymbol{C}_e^{1/2}$ the Cholesky decomposition of $\boldsymbol{C}_e$. After $\boldsymbol{H}$ is found, the estimate of $\boldsymbol{\xi}$ can be obtained by

$$
\hat{\boldsymbol{\xi}} = \boldsymbol{H}\boldsymbol{b}. \quad (18)
$$

As can be seen from (6), the elements of $\boldsymbol{\xi}$ are in fact not independent of each other and the estimate of $\boldsymbol{\xi}$ can be refined by exploiting the relationship between elements of $\boldsymbol{\xi}$, which can be represented by

$$
\boldsymbol{G}\boldsymbol{\omega} = \boldsymbol{\xi}, \quad (19)
$$

where $\boldsymbol{\omega} = [\theta_1, \theta_2, \boldsymbol{x}^T]^T$, and

$$
\boldsymbol{G} = \begin{bmatrix} \tilde{\boldsymbol{G}} \\ \boldsymbol{I}_4 \end{bmatrix}, \quad \tilde{\boldsymbol{G}} = \frac{1}{2}\begin{bmatrix} \theta_1 & 0 & 0 & 0 \\ 0 & \theta_2 & -\frac{x}{c^2} & -\frac{y}{c^2} \\ \theta_2 & \theta_1 & 0 & 0 \end{bmatrix}. \quad (20)
$$

The refinement of the estimate of $\boldsymbol{\xi}$ is carried out by

$$
\hat{\boldsymbol{\omega}} = (\hat{\boldsymbol{G}}^T\hat{\boldsymbol{G}})^{-1}\hat{\boldsymbol{G}}^T\hat{\boldsymbol{\xi}}, \quad (21)
$$

where $\hat{\boldsymbol{G}}$ is obtained by putting the estimated values of x, y, θ_1 and θ_2 from $\hat{\boldsymbol{\xi}}$ of (18) into the corresponding variables in $\boldsymbol{G}$. The final estimates of θ_s and θ_0 are obtained from the estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ in (21) by $\hat{\theta}_s = 1/\hat{\theta}_1$ and $\hat{\theta}_0 = \hat{\theta}_2/\hat{\theta}_1$.

5. SIMULATION RESULTS

In the simulations, there are three anchors located at $(1,2)$, $(10,3)$, and $(4,11)$, with unit of meter. The sensor node to be located and synchronized is located at $(5,6)$. The number of time-stamp exchange round, clock skew and clock offset are set to $M = 2$, $\theta_s = 1.005$ and $\theta_0 = 50$ns, respectively. The unit of all time-stamps in the two-way message exchange is nanosecond. The uncertainties Δa_{xl} and Δa_{xl} are uniformly drawn from $[-0.01, 0.01]$, while $\Delta\theta_{sl}$ and $\Delta\theta_{0l}$ from $[-0.005, 0.005]$. The ρ_A and ρ_C are calculated accordingly from (11). Because the norm of $\boldsymbol{\xi}$ is unknown, N is estimated from the LS solution $\hat{N} = \|\hat{\boldsymbol{\xi}}_{LS}\|$.

Figures 2 and 3 show the MSE of location estimation, which is defined as $E\{(x-\hat{x})^2 + (y-\hat{y})^2\}$, and MSE of time skew and time offset estimation, respectively. It can be

seen that the MSE performances of the proposed robust minimax estimator is better than the conventional LS estimator, especially when the variance of TOA detection error σ_n^2 is large. When the TOA detection error is small, LS and the robust minmax provide similar performances.

6. CONCLUSIONS

In this paper, we proposed a unified framework to jointly solve the localization and time synchronization problems with bounded uncertainties in the anchor locations and timings. The problem was formulated into a linear equation with errors in the model matrix and a robust minimax estimator was presented. Simulations showed that the robust minimax estimator performs better than conventional LS estimator, especially when the variance of TOA detection error is large. However, considering both performance and computational complexity, the LS estimator might be preferable when the variance of TOA detection error is small.

7. REFERENCES

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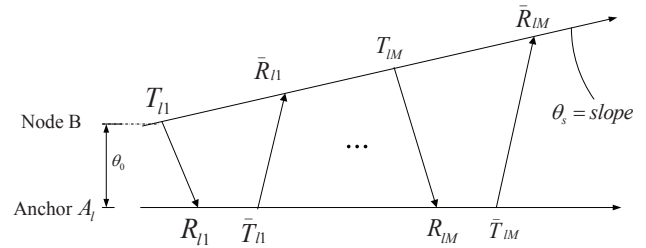
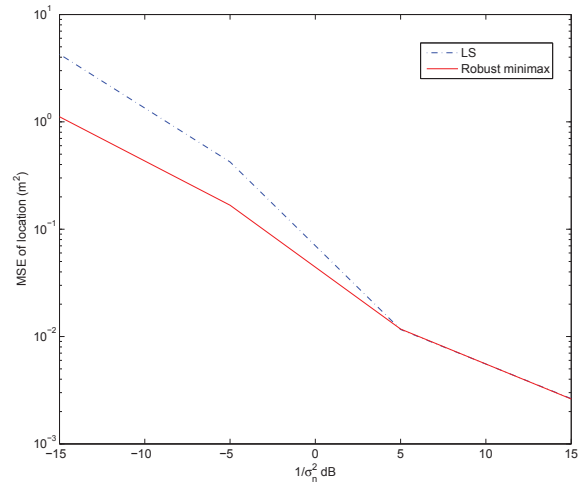**Fig. 1**. Two-way time-stamp exchange between node B and the lth anchor A_l.

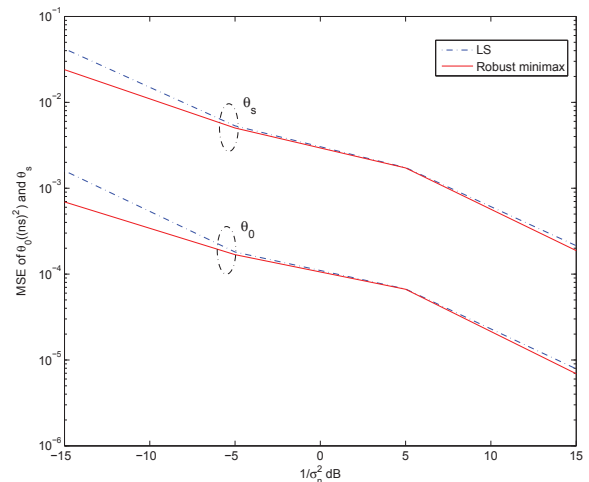**Fig. 2**. MSE of location versus variance of TOA detection error $1/\sigma_n^2$.

Fig. 3. MSE of θ_s and θ_0 versus variance of TOA detection error $1/\sigma_n^2$.