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A model of gas buildup and release in crater lakes

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Abstract. The sudden release of carbon dioxide gas, which may accumulate gradually within the stratified water bodies of a crater lake, caused two fatal disasters in Cameroon during the past decade. We model the various processes that may have led to the release by considering (1) gas injection, (2) an internal triggering mechanism, (3) propagation of a disturbance after triggering, and (4) the forces that end the outburst. We suggest that the 1986 Lake Nyos outburst was triggered at the lake bottom. The final explosive stage of the release ended quickly when a sufficiently large volume of gas bubbles prevented circulation of water eddies within the lake. A cellular automaton model is used to estimate the amount of carbon dioxide gas released and the characteristic time interval between successive outbursts. If both the gas accumulation rate and the diffusion rate of carbon dioxide through water are constant, then rapid gas release will occur at fairly regular intervals. In which case, the amount of gas released from Lake Nyos is 0.17 ± 0.05 km^3 at standard temperature and pressure each 37 ± 10 years. It is possible, however, that an external event could trigger a sudden release or that the diffusion rate of carbon dioxide may change as more gas accumulates, which could shorten the release time.

Introduction

Twice in the mid-1980s the people in Cameroon suffered two unusual natural disasters. Both had fatal consequences. The first occurred in Lake Monoun on August 15, 1984, and claimed 37 lives [*Sigurdsson et al.*, 1987]. The second occurred in Lake Nyos on August 21, 1986, and claimed about 1700 lives [*Kling et al.*, 1987]. These fatal disasters were probably caused by the rapid release of CO_2 gas dissolved within the water of crater lakes. Obviously, an understanding of the possible triggering and gas release mechanisms may prevent future casualties.

We propose a gas release mechanism based on a cellular automaton model that describes the buildup and release of dissolved CO_2. If both the gas influx rate and the diffusion of CO_2 through the water are constant, then the time between successive releases and the amount of gas released are regular, as long as no external event, such as a landslide into the lake, triggers a gas outburst. Through the calculations are specific to Lake Nyos, they can be applied to other crater lakes.

Previous Work for Crater Lake Outbursts

The results of early research into the Nyos disaster are summarized in the proceedings of an International conference in March 1987 [see, *Sigurdsson et al.*, 1987; *Tazieff et al.*, 1987]. Estimates of the released volume of CO_2 at standard temperature and pressure (STP) range from 0.12 km^3 to 1.2 km^3 [*Kling et al.*, 1987; *Tietze*, 1987, 1992; *Kusakabe et al.*, 1989; *Giggenbach*, 1990]. The range of a factor of 10 of the released volume is a result of using various assumptions about the degree of saturation of lake water by CO_2.

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Later investigations of Lake Nyos found a significant and alarming influx of CO_2 [*Nojiri et al.*, 1990, 1993; *Evans et al.*, 1993, 1994]. Thus the CO_2 eruption is likely to be limnic rather than phreatic. *Nojiri et al.* [1990] estimated that the stagnant bottom water, which is at depths below 30 m at Lake Nyos, will saturate with CO_2 within 50 years. *Evans et al.* [1994] estimated a period of 20 years to saturation when they considered the change in chemical composition of Lake Nyos during the 6 years after the fatal event.

The lack of reactive sulphur and chlorine compounds in Lake Nyos and the old age of the CO_2 in the lake, deduced from ^{14}C dating, indicate that the CO_2 cannot have a volcanic or biogenic origin. The general consensus is that the gas is of magmatic origin [*Kling et al.*, 1987; *Kusakabe et al.*, 1989, 1992; *Nojiri et al.*, 1990, 1993; *Sigurdsson et al.*, 1987; *Tietze*, 1987, 1992]. The triggering and gas release mechanism remains unknown. Proposed triggering mechanisms include a landslide into the lake, seismic shaking, vertical movement of water caused by internal waves, and inflow of cold surface water. An earthquake-induced landslide was recorded before the 1984 Monoun event. In addition to causing a disturbance of the lake, a landslide may disperse fine particles, which could facilitate the nucleation of CO_2 bubbles [*Sigurdsson et al.*, 1987]. No earthquake, however, occurred near Lake Nyos on the day of the 1986 outburst. Furthermore, trigger by a landslide should produce frequent, small-scale gas outbursts, because there are more small than large landslides into the lake. A distribution in volume of outburst is not reported. Since both the Monoun and Nyos events occurred in August, which is the coolest and rainiest month of the year in Cameroon, the inflow of cold water into a lake has been proposed as a triggering mechanism [*Giggenbach*, 1990; *Kling et al.*, 1987]. *Giggenbach* [1990] argued that a vertical temperature difference of only 2.0°C near the lake surface would be enough to induce a convective overturn. *Kusakabe et al.* [1989], however, calculated that the inflow of water had to be cooler than 11°C to cause the release of only one third of the CO_2 dissolved in the lake. Such a low temperature is impossible in the climate of Lake Nyos.

Circumstantial evidence suggests that the gas release was confined to a small area of Lake Nyos. The wave damage to the shoreline caused by the gas release was highly asymmetric, suggesting gas release from a localized area of the lake surface. If the lake was well mixed by the gas outburst, then it would lose all its stratification. Contrary to this, *Kanari* [1989] and *Tietze* [1987], who conducted physical and chemical surveys of Lake Nyos around 50 days after the outburst, reported high concentrations of dissolved CO_2 and iron, which contribute to stable stratification of lake water. They suggested that most of the gas was released from a small area of the lake, and so the lake stratification was largely unaffected.

Tietze [1987, 1992] developed the "fountain limnic" model for the gas outbursts. According to his model, gas release began at a shallow or intermediate-level chemocline that has a steep CO_2-concentration gradient when the gas release was triggered by pressure reduction, perhaps by an internal wave. In this model, gas release migrated downward steadily, involving deeper and deeper water layers in a fountain-like fashion. Eventually, fountaining could not sustain the deepening process, and so it stopped, leaving water below about 150 m unaffected. A shallow initiation depth was proposed also by a model in which gas release proceeded laterally over the lake surface [*Giggenbach*, 1990]. In this case, minor gas loss and mixing occurred mostly above 100 m depth.

In a review of earlier results, *Evans et al.* [1994] found that disruption of preexisting stratification was more extensive than previously proposed; hence even the deepest water layers were involved in the event at Lake Nyos. They assumed that the release began at the base of a 50-m-deep chemocline, analogous to *Tietze* [1987, 1992], and that the gas loss was continuously confined to a small area, analogous to *Kanari* [1989] and *Tietze* [1987]. The gas release was a two-phase process. Phase one was a relatively nonviolent release of gas, which involved only the chemocline above 50 m. In response to some trigger, an area of this chemocline was forced upward, and gas release began. Continuous release caused this water to rise to the surface, where gas was released to the atmosphere nonviolently. The base of the water column deepened slowly as the gas release continued. When the column base reached a secondary chemocline, gas release was accelerated because CO_2 concentration was high in in the lower chemocline. This marked the beginning of phase two, when the process became violent and noise from the lake was noticed [*Kling et al.*, 1987].

In summary, CO_2 injected slowly at the lake bottom caused an increase in CO_2 concentration in the lake. The sudden outburst is probably triggered by internal vertical water instability, instead of an external trigger, such as an earthquake. The area of gas release was confined to a small surface that involved deep water layers.

A New Gas Outburst Scenario

Gas Buildup

We propose a model of gas buildup in only the vertical dimension, where x is the distance measured from the bottom of the lake upward. This model includes the basic elements of gas buildup and is simple enough to compute analytically. Gas is injected at a uniform rate R at the bottom of the lake. The effective (vertical) diffusion constant D of CO_2 is assumed to be constant at all depths and at all time. Water density is also assumed to be constant at all depths. On the basis of these assumptions, the concentration of CO_2, called ρ, is given by

$$\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2} + R\delta(x), \tag{1}$$

where $\delta(x)$ is the Dirac delta function with $\delta(x) = 0$ if $x \neq 0$ and $\int_{-\infty}^{+\infty} \delta(x)dx = 1$. The boundary conditions for this system are $\partial\rho/\partial x = 0$ at $x = 0$ and $\rho = 0$ at $x = \ell$, where ℓ is the depth of the lake.

Applying the Green's function method to solve (1), we get

$$\rho(x,t) = \int_0^t \int_0^\ell G(x,t;x',t')\rho(x',t')dx'dt', \quad (2)$$

where $G(x,t;x',t') = 0$ if $t < t'$, and

$$\begin{aligned} G(x,t;x',t') &= 2\sum_{n=-\infty}^{+\infty} \cos\frac{(2n+1)\pi x}{2\ell}\cos\frac{(2n+1)\pi x'}{2\ell} \\ &\quad \exp\left[-\frac{(2n+1)^2\pi^2 D(t-t')}{4\ell^2}\right] \\ &\approx \frac{1}{2\sqrt{\pi D(t-t')}}\left[e^{-(x-x')^2/4D(t-t')}\right. \\ &\quad \left.+e^{-(x+x')^2/4D(t-t')}\right] \end{aligned} \quad (3)$$

if $t \geq t'$. The above approximation is obtained by replacing the infinite sum by an integral. Combining (2) and (3), we have

$$\begin{aligned} \rho(x,t) &\approx \int_0^t \frac{R}{\sqrt{\pi D t'}} e^{-x^2/4Dt'}dt' + \int_0^{+\infty}\frac{\rho(x',0)}{2\sqrt{\pi D t}} \times \\ &\quad \left[e^{-(x-x')^2/4Dt} + e^{-(x+x')^2/4Dt}\right]dx', \end{aligned} \quad (4)$$

where $\rho(x',0)$ is the initial distribution of CO_2 in the lake. Provided that $\rho(x,0) = 0$, the concentration of CO_2 at the lake bottom is given by

$$\rho(0,t) = 2R\sqrt{\frac{t}{\pi D}}\,. \quad (5)$$

Thus the rate of increase in CO_2 concentration at the lake bottom slows down gradually. This trend is also true if $\rho(x,0)$ is nonzero. This result is consistent with a measured slowing in the rate of increased CO_2 concentration at the bottom of Lake Nyos after 1987 [*Evans et al.*, 1993].

Triggering of Gas Release

On the basis of our earlier discussion, the triggering is likely to be an internal process caused by the oversaturation of CO_2 in the lake. We assume that an outburst is triggered whenever the local CO_2 concentration equals its saturation value, even though the gas release may sometimes redissolve into some water layers on top. If CO_2 is injected from the lake bottom, it usually takes place near a chemocline or at the lake bottom [cf. *Tietze*, 1987, 1992]. There existed three chemoclines before the 1986 outburst with estimated depths about 50 m (upper), 150 m (intermediate), and 190 m (lower) [see, e.g., *Evans et al.*, 1994 and *Tietze*, 1987]. We model the chemoclines by approximating the diffusion constant by a piecewise constant function of depth in the form

$$\begin{aligned} D(x) &= D_1 && \text{if } x < L_1 \\ D(x) &= D_2 && \text{if } L_1 \leq x < L_2 \\ D(x) &= D_3 && \text{if } L_2 \leq x < L_3 \\ D(x) &= D_4 && \text{if } x \geq L_3, \end{aligned} \quad (6)$$

where x is the vertical distance from the lake bottom $L_1 = 20$ m, $L_2 = 60$ m, and $L_3 = 160$ m are the distances of the lower, intermediate, and upper chemocline from the lake bottom, respectively. The solution of our driven diffusion equation (1) is

$$\begin{aligned} \rho(x_r,t) &\approx \int_0^t \frac{R}{\sqrt{\pi D_1 t'}} e^{-x_r^2/4D_1t'}dt' + \\ &\quad \int_0^{+\infty}\frac{\rho(x_r',0)}{2\sqrt{\pi D_1 t}}\left[e^{-(x_r-x_r')^2/4D_1t}\right. \\ &\quad \left.+\, e^{-(x_r+x_r')^2/4D_1t}\right]dx_r', \end{aligned} \quad (7)$$

where

$$\begin{aligned} x_r &= x && \text{if } x < L_1 \\ x_r &= L_1 + \sqrt{\tfrac{D_1}{D_2}}(x-L_1) && \text{if } L_1 \leq x < L_2 \\ x_r &= L_1 + \sqrt{\tfrac{D_1}{D_2}}(L_2-L_1) + \\ &\quad \sqrt{\tfrac{D_1}{D_3}}(x-L_2) && \text{if } L_2 \leq x < L_3 \\ x_r &= L_1 + \sqrt{\tfrac{D_1}{D_2}}(L_2-L_1) + \\ &\quad \sqrt{\tfrac{D_1}{D_3}}(L_3-L_2) + \\ &\quad \sqrt{\tfrac{D_1}{D_4}}(x-L_3) && \text{if } x \geq L_3. \end{aligned} \quad (8)$$

To determine the triggering location of degassing, we compare the value of $\rho(x,t)$ with its saturation value $\rho_{sat}(x)$ at various points x. Suppose T is the time when CO_2 concentration at the lake bottom reaches the saturation value, i.e., $\rho(0,T) = \rho_{sat}(0)$. We estimate the concentration of CO_2 at all the chemoclines at the same time T. If $\rho(L_i,T)$ for $i = 1, 2$, and 3 are all less than their corresponding CO_2 saturation concentrations, then water at the chemocline is still unsaturated when the bottom of the lake becomes unstable. In this case, the lake bottom spontaneously degases first. On the other hand, if either one of the $\rho(L_i,T)$ is greater than $\rho_{sat}(L_i)$ for $i = 1, 2$, or 3, then water at the chemocline becomes unstable sooner than that at the lake bottom. So spontaneous degassing will be triggered at the chemocline.

Unfortunately, we are unable to perform the integration in (7) analytically. So we put in the appropriate numbers and perform the integration numerically. The CO_2 injection rate is about 2.5×10^8 mol yr^{-1} [*Evans et al.*, 1994], and the area of the lake basin is about 0.5 km^2 [*Nojiri et al.*, 1993], so that $R \approx 500$ mol m^{-2} yr^{-1}. From the CO_2 concentration curves of *Tietze* [1987] and *Evans et al.* [1993], D_1, D_2, D_3, and D_4 are estimated to be about 40 m^2 yr^{-1}, 130 m^2 yr^{-1}, 20 m^2 yr^{-1}, and 500 m^2 yr^{-1}, respectively. The $\rho(x_r', 0)$ curve can be estimated from the measurements made by *Tietze* [1987] in October/November 1986, a few months after the gas outburst, and ρ_{sat} is deduced from *Houghton et al.* [1957]. We solve (7) numerically and find that the characteristic time between two outbursts T is about 37 ± 10 years, assuming that gas outbursts are triggered internally in the lake bottom. Our finding is consistent with the recharge time estimates by *Evans et al.* [1994], *Kling et al.* [1994], and *Nojiri et al.* [1993], which are 37 years, 40 years, and 20 years, respectively. All four estimates are not inconsistent with each other since substantial approximations are involved in computing each estimate.

By performing numerical integration of (7), $\rho(x, T)$ at different x can be found. As shown in Table 1, the lake bottom reaches the saturation point first. Therefore we believe that degassing begins at the bottom rather than at the chemocline of Lake Nyos (see Figure 1a). Since the major errors in the above analysis come from the possible time variation of the effective diffusion constant D and the uncertain initial CO_2 concentration curve, our finding is tentative. Nevertheless, degassing from the lake bottom seems likely because a slightly different value of D gives the same result. Finally, we remark that degassing starting from one of the chemoclines is possible in other lakes because the values of R and D and the depth of chemoclines vary greatly from one lake to another.

Table 1. CO_2 Concentration in Lake Nyos When the Lake Bottom Is Just Saturated

	Upper Chemocline	Intermediate Chemocline	Lower Chemocline	Lake Bottom
d[2]	50	150	190	210
ρ_{sat}[3]	160	450	555	630
ρ_T[3]	40 ± 30	290 ± 100	450 ± 150	630

The saturation time T is about 37 years.
[2]Depth of the layer d in meters.
[3]CO_2 concentration at saturation ρ_{sat} and when the lake bottom is just saturated ρ_T for different layers in units of moles per cubic meter.

Disturbance Propagation and Region of Degassing

Now we address the disturbance propagation. Although the exact hydrodynamic equations describing the gas release are difficult to solve, the speed of disturbance propagation can be estimated. The Navier-Stokes equation for incompressible fluid is

$$\frac{\partial \vec{v}}{\partial t} = \vec{v} \cdot \nabla \vec{v} + \nu \nabla^2 \vec{v} + \frac{1}{\rho_w} \vec{F} - \frac{1}{\rho_w} \nabla P, \qquad (9)$$

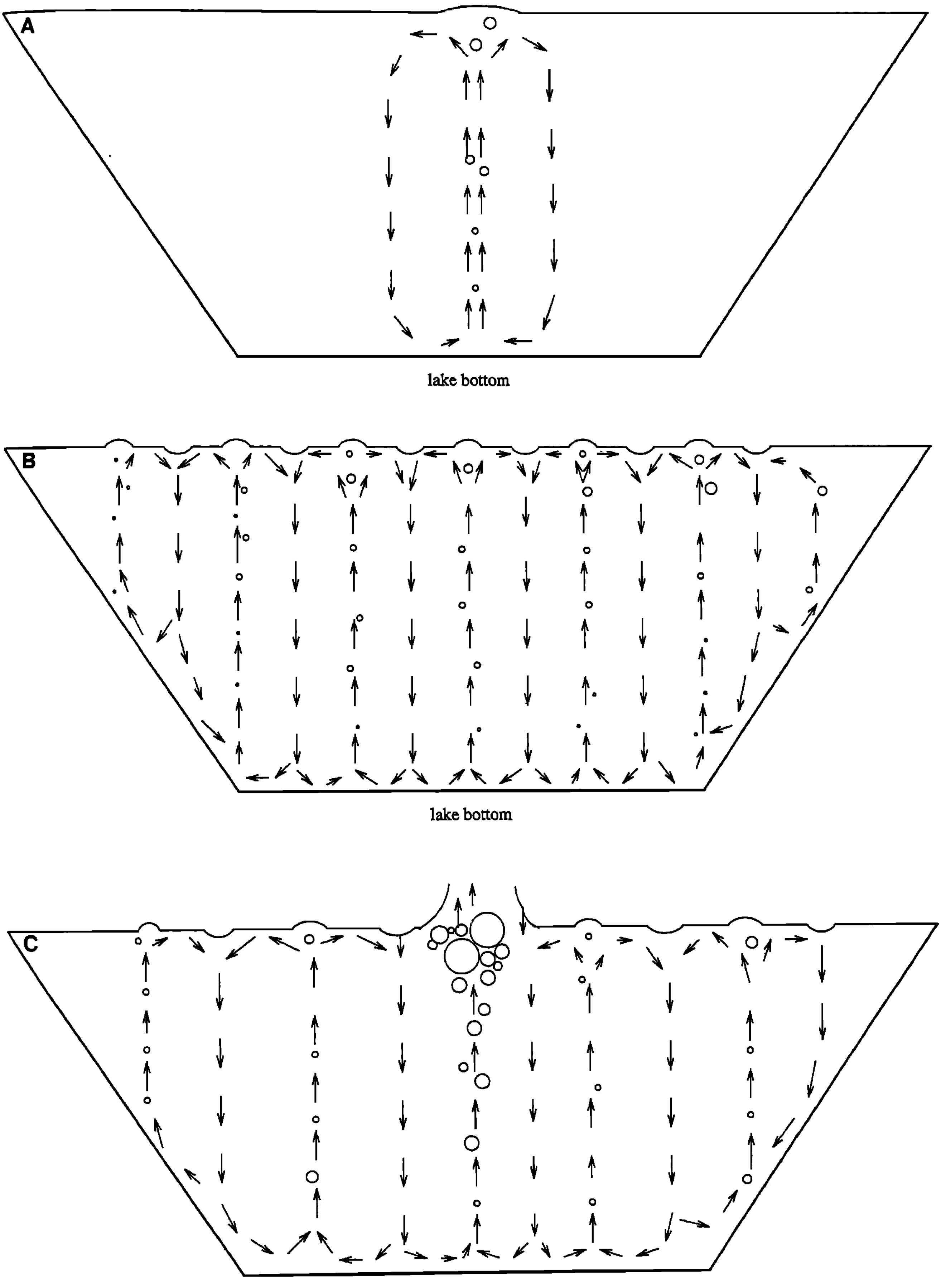
where $\vec{v}$ is the velocity of the fluid, $\vec{F}$ is the force per unit volume acting on the fluid, P is the pressure, ρ_w is the density, and ν is the kinematic viscosity (whose contribution can be neglected when studying lakes). Consider a CO_2 gas bubble of radius λ that formed near the boundary of the gas release region. Eventually, the gas bubble will rise with a terminal velocity v_{term}, and the water around it will be dragged upward with a velocity $\lesssim v_{term}$. Now consider the water at a point $\vec{x}$ with horizontal distance d ($\gtrsim \lambda$) from the bubble. The advection term $\vec{v} \cdot \nabla \vec{v} \approx v_{term} \vec{v}(\vec{x})/d$ is the dominant term in (9). Thus the characteristic timescale for the velocity of water at $\vec{x}$ to rise is $\approx d/v_{term}$, and so the characteristic disturbance propagation speed of the disturbance (not the water flow speed) is $\lesssim d \times (d/v_{term})^{-1} = v_{term}$.

The value of v_{term} has been investigated experimentally by *Davis and Taylor* [1950]. For air bubbles rising through water at room temperature, the terminal velocity is approximated by

$$v_{term} \approx \frac{2}{3}\sqrt{g\lambda}, \qquad (10)$$

where g is the acceleration due to gravity. During the initial stage of the gas outburst, a gas bubble will be small and so will its ascent velocity. For example, a bubble with a radius of 1 mm will have a terminal velocity of about 0.06 m s^{-1}. As more gas is released and as the gas bubbles rise and merge with each other, their radius can easily reach 10 cm or so. Then the disturbance propagation speed rises to about 0.7 m s^{-1}. It is unlikely to have gas bubbles much larger than 10 cm in radius because they are unstable and will quickly break up into smaller bubbles [*Kanari*, 1989]. Therefore the vertical disturbance propagation speed is about 0.7 m s^{-1}. Since the rising gas bubbles are roughly spherical, the horizontal disturbance propagation speed is at most ≈ 0.7 m s^{-1}. Further discussions on the shape of gas bubbles and the thermal physics of rising gas bubbles can be found in *Kanari* [1989]. So, for Lake Nyos, which is about 2 km wide, horizontal propagation of a disturbance may take $\gtrsim 50$ min (see Figure 1a). Since the size of a gas bubble at the lake bottom may not be large, the disturbance propagation speed there should

Figure 1. Schematic representation of the degassing process in Lake Nyos. Arrows represent the directions of water flow, and circles represent the CO_2 gas bubbles. (a) Shortly after the triggering of outburst at the lake bottom, degassing is confined to a small area because the disturbance propagation speed is slow. (b) Later, when the disturbance has propagated throughout the lake, the water eddies organize into a fairly regular and stable Bénard roll-like structure. Degassing takes place all across the lake although the total area of degassing on the lake surface is small. At this stage, degassing is nonviolent and may take an hour or more. (c) Finally, large gas bubbles are formed which induce a percolation transition at some portion of the lake surface leading to a quick termination of degassing. Degassing becomes violent. This stage lasts for about a minute.


A
lake bottom
B
lake bottom
C
lake bottom

be slower. Such a differential disturbance propagation may favor the idea of gradual deepening of vortices [see, e.g., *Tietze*, 1992].

Equation (10) also indicates that the degassing process may occur as two phases. The first phase is a relatively mild degassing with small gas bubbles. The disturbance propagation velocity is low in this stage, and so nonviolent degassing can take an hour or so. Eventually, the gas bubbles become so large that a rapid disturbance propagation occurs. This is the second phase, which is likely to be explosive. In this respect, our degassing model is remarkably similar to the two-stage degassing scenario proposed by *Evans et al.* [1994] recently. The disturbance propagation time is a measure of how fast a local instability caused by oversaturation of CO_2 can propagate through the lake. The total degassing time, however, measures the total time elapsed during the gas outburst. Therefore our estimate of a disturbance propagation time of approximately an hour is consistent with the degassing time proposed by *Evans et al.* [1994], which may take up to a few hours.

Next, we come to the question of how large the gas release area is. Obviously, the degassing region coincides with the location on the lake surface where water moves upward. The previous models suggest a small degassing region, either the convective region is confined to a small volume [*Tietze*, 1987] (which is unlikely to be the case since the CO_2 gas released would not be enough to account for the accident) or the horizontal size of a typical eddy near the lake surface extends over the whole lake, which is about 2 km for Lake Nyos [*Evans et al.*, 1994]. As compared to the depth of the lake, which is about 210 m, the eddy has to be highly elongated in the horizontal direction.

Because the onset of turbulence is driven by an upward moving CO_2 gas bubble, the eddies are likely to be elongated vertically instead of horizontally. There are two possibilities. First, there is a single eddy with limited horizontal extension. Since the degassing time is at most a few hours [*Evans et al.*, 1994], which is too short for horizontal transport of CO_2 to be significant, the amount of CO_2 gas released this way is not large enough to cause the fatal accident. Alternatively, the propagation of disturbance may eventually set up a large-scale vertically elongated eddy current in the lake. This scenario does not contradict the small gas release area on the lake surface because eddies may form stable structures, such as the well-known Bénard rolls formed by uniformly heating a fluid from the bottom [*Bénard*, 1901]. In fact, the formation of regular stable convective structure is commonly observed when fluid just becomes convectively unstable. The gas release area, which equals the area of upward moving fluid near the lake surface, can be as small as a few percent as compared to the surface area of the lake. In this way, the vertical velocity of fluid is small in most parts of the lake, and hence the stable stratification of the lake is largely unaffected during the gas release [*Kanari*, 1989; *Tietze*, 1987] (see Figure 1b). It is interesting to see how fast the convectively unstable lake water can organize itself into structures similar to Bénard rolls. However, this question goes beyond the limit of simple order of magnitude estimations, and a full-scale hydrodynamic simulation is required to give us the answer.

End of an Outburst

After careful examination of the report of an eyewitness, *Evans et al.* [1994] suggest that the final explosive stage of the gas outburst in Lake Nyos lasted less than a minute. Here we provide a simple mechanism for the quick end of gas outburst.

In the absence of gas bubbles, the motion of water can be described by the Navier-Stokes equation. The Navier-Stokes hydrodynamics description, however, is still valid when the volume of gas bubbles is small compared to the volume of water. Nevertheless, the boundary conditions for the lake water are complicated because the boundaries between water and the CO_2 gas bubbles must be included. If the volume of gas bubbles in water becomes large, the geometry of the water system changes dramatically. In that case, gas bubbles occupy most of the volume so that the flow of water at distant regions is disconnected. This phenomenon is similar to the percolation transition as one varies the conductance probability in a random electrical network [*Stauffer and Aharony*, 1992]. At this point, the usual hydrodynamic description of water will break down. Each disjoint portion of water will move according to gravity and the gas pressure will act locally. The local flow of water and gas is temporarily decoupled from the rest of the water in the lake.

We suggest that the gas-bubble concentration during the final stage of the gas outburst near some portion of the lake surface was so high that percolation transition occured and the hydrodynamic approximation failed. Thus the global circulation of water eddies close to the lake surface is quickly and strongly suppressed, leading to a quick end of the final phase (see Figure 1c).

Our gas-outburst scenario is summarized in Figure 1. The gas release, at least for Lake Nyos, probably begins at the lake bottom. The disturbance takes at least an hour to propagate throughout the entire lake. Degassing at this stage may not be violent. Water eddies are formed by the upward moving CO_2 gas bubbles. They will form quickly as regular and stationary rolls. The degassing area remains small, and the stratification is unaffected. The speed of the stationary vortex rolls increases as the size of the CO_2 gas bubbles increases, degassing becomes more and more violent. Eventually, percolation transition occurs near the lake surface, which will stop the eddy circulation. This ends the final, rapid degassing process.

The Cellular Automaton Model

Without any effective mechanism to discharge CO_2 gas continuously from a gas-rich crater lake, gas outbursts will be repetitive [*Tietze*, 1987, 1992]. Thus it is useful to investigate the statistics of the amount of gas released and the time interval between successive outbursts. Should successive gas outbursts be regular or chaotic? Should the spatial fluctuation of CO_2 in the lake be important? To answer these questions, we have to go beyond the simple order of magnitude estimations done in the previous section. We use a simple cellular automaton model, which is consistent with our revised degassing scenario, to examine these questions.

Description of the Model

We approximate the lake bottom by a $K \times K$ square grid with close boundary conditions. Our results are insensitive to the choice of the shape of the lake. We label the grid points by (i, j) with $1 \leq i$ and $j \leq K$. The mass of CO_2 that dissolves in the vertical column of water at the grid point (i, j) is denoted by m_{ij}. The vertical diffusion of CO_2 is relatively slow. Also, our analysis in the previous section suggests that the degassing of Lake Nyos probably begins at the lake bottom. These points support the use of a two-dimensional cellular automaton model.

We simulate the effects of spatial fluctuation of CO_2 injection, which is neglected in all previous investigations, by making a small amount of CO_2 with mass Δm dissolve in an arbitrarily chosen grid point in each small time interval Δt. The value of Δm is chosen to be the mass of a typical smallest unit of CO_2 injected into the lake water, called a CO_2 quantum. The experimentally observed CO_2 injection rate equals $\Delta m/\Delta t$. Therefore Δm and Δt are not independent variables. Once Δm is known, Δt is also determined accordingly. Furthermore, the value of K, which measures how fine our grid is, cannot be randomly chosen. First, K cannot be too large because the size of a grid point cannot become smaller than the size of a typical CO_2 mass quantum; otherwise, the injection of CO_2 into exactly one grid point each time cannot be justified. However, K cannot be too small; otherwise, the finite size effects becomes important.

We model the horizontal motion of water by means of an effective (horizontal) diffusion constant D_{eff} on the two-dimensional grid. Gas release is triggered whenever the value of m_{ij} in a grid point exceeds a fixed threshold m_{sat} determined by the saturation concentration of CO_2. This is a natural condition for the spontaneous formation of a CO_2 gas bubble. "Premature" gas release is possible whenever there is strong external trigger, such as an earthquake, setting up a large vertical water movement. However, we do not pursue this possibility because of the lack of conclusive data supporting the external trigger hypothesis. After an outburst, the mass of CO_2 remaining in the triggering grid point is reduced to m_{res} determined by the scale and the rate of circulation in water. To simplify matters, we assume m_{res} to be a fixed constant.

We simulate propagation of the disturbance by requiring all the neighboring grid points around the triggering location to release gas provided that their CO_2 content is greater than m_{prop}. This is reasonable provided that $m_{\text{sat}} \gg m_{\text{prop}} \gg m_{\text{res}}$. The propagation process is repeated until no more new grid points releases gas. Since the gas release rate is much larger than the gas injection rate, we assume that the processes of disturbance propagation and gas release end within a single time step Δt, a simplification that is sufficient unless one wants to study the temporal profile of a gas outburst in detail.

The CO_2 content of those grid points turning convectively unstable is set to m_{res} after the outburst. After a sufficiently long time, the CO_2 content of any grid point is greater than or equal to m_{res}. Moreover, the value of m_{res} does not affect the statistics of the outburst. So, for simplicity, we set $m_{\text{res}} = 0$ in our numerical simulation.

The flow chart of our numerical simulation is shown in Figure 2. The processes of gas introduction and occasional gas release when the triggering condition is satisfied are repeated until the system has reached an equilibrium state characterized by the balance of the time-averaged gas injection and gas ejection rates. Statistics of the amount of CO_2 gas release and the time between successive gas outbursts are then taken. The long-term behavior of the model is found to be independent of the initial conditions. The model we have studied here belongs to a very general class of cellular automaton model [*Chan et al.*, 1995a], which has been used to study the burst statistics of astrophysical type I X ray bursters [*Chau et al.*, 1995a, b].

Numerical Results

We choose $K = 64$ in our numerical simulation. In the case of Lake Nyos, every grid point is a square of side ≈ 30 m wide. This choice of K is reasonable: it is large enough to avoid the finite size effect of the grid and small enough to make the assumption of adding CO_2 to a single grid point each time valid. The total gas injection rates $\Delta m/\Delta t$ are taken to be 0.28 kg s^{-1} for Lake Nyos and 0.024 kg s^{-1} for Lake Monoun [*Kling et al.*, 1994].

To examine the effects of diffusion, spatial fluctuation in CO_2 injection, and the finite size effect of typical CO_2 mass quantum, we fix $m_{\text{sat}} - m_{\text{res}} = 500\ \Delta m$, $m_{\text{prop}} - m_{\text{res}} = 50\ \Delta m$, and choose $D_{\text{eff}} = 1.7 \times 10^{-6}$ and 1.7×10^{-3} m^2 s^{-1} in the following simulations. Computational time increases roughly linearly with the product of D_{eff} and $(m_{\text{sat}} - m_{\text{res}})/\Delta m$, which forbids us to perform numerical experiments with a large value of $(m_{\text{sat}} - m_{\text{res}})/\Delta m$ and D_{eff}. Figure 3 shows a strong positive correlation between the amount of CO_2 released (called the burst size s) and the time since last outburst t_{last}. Moreover, the standard deviation of t_{last} and s decrease with increasing D_{eff}, indicating that the system behaves more regularly with increasing D_{eff}. The typical outburst size is $2.0 \times 10^6\ \Delta m$ when $D_{\text{eff}} = 1.7 \times 10^{-3}$ m^2 s^{-1}. The simulation shows that the amount of CO_2 injected into the lake since its last outburst will be released completely in the coming event. Besides, both s and t_{last} show fairly regular behaviors suggesting that both the time and size of

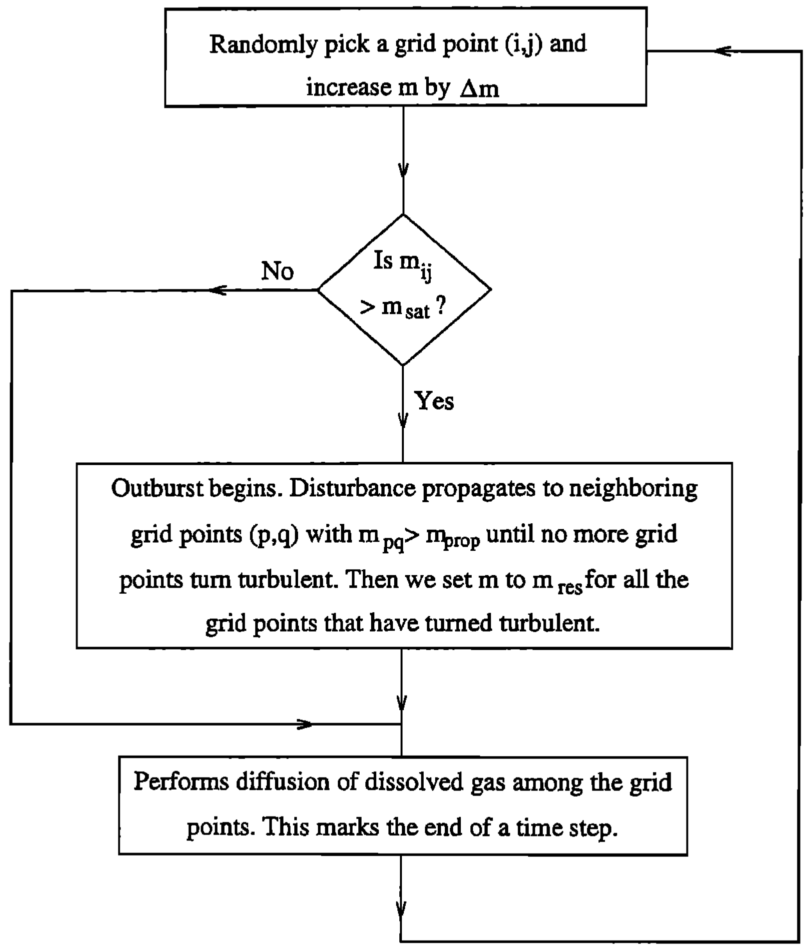


Figure 2. Flow chart of the numerical simulation program.

the outburst are predictable if physical parameters at the lake boundaries (such as influx rate) remain constant. Thus the effects of spatial fluctuation of CO_2 and finiteness of CO_2 mass quanta are not important for Lake Nyos (and possibly also for other gas-bearing crater lakes).

Actually, the outburst statistics can be explained qualitatively [*Chan et al.*, 1995a; *Chau et al.*, 1995a]. A "burnable cluster" is defined as a collection of grid points such that all of them will degas via disturbance propagation whenever any one of them becomes oversaturated and degases. Regular outbursts are the result of a single large burnable cluster covering almost the entire lake just before a gas outburst. This is possible when (1) the propagation threshold (which equals $m_{\text{prop}} - m_{\text{res}}$) is much less than the triggering threshold (which equals $m_{\text{sat}} - m_{\text{res}}$), or (2) the diffusion constant D_{eff} is so large that fluctuation of CO_2 concentration is smoothed out [*Chan et al.*, 1995a]. This explains why the regularity of the outburst increases with increasing D_{eff} (see Figure 3). Finally, the arguments leading to the above two conditions resulting in regular outbursts are completely general and are applicable to any griding method (such as dividing the lake into regular hexagons instead of squares) and lake morphology.

On the other hand, if the propagation threshold is comparable to the triggering threshold and the effective diffusion constant is small, a collection of small burnable clusters (instead of a single large one) appears just before an outburst. In this case, the gas release will differ from one outburst to another. The time between successive outbursts also becomes irregular and unpredictable [*Chan et al.*, 1995a].

Equating the time between successive outbursts obtained in the simulation (which equals $2.0 \times 10^6 \Delta t$) to our estimated 37-year recurrence time obtained in the previous section, the time elapsed in each cellular automaton time step Δt is about 9.7 min. Given that the degassing takes approximately an hour, our assumption of degassing the system in a single time step is reasonable. We may modify the cellular automaton model so that degassing takes more than one time step. Because of the wide separation of the gas outburst time and the recurrence time, however, the observed burst statistics are not sensitively dependent on the finite degassing time.

The mass of a typical CO_2 quantum in this simulation Δm can be deduced from the gas injection rate and is equal to 160 kg. Thus the typical mass of gas ejected in an outburst equals 3×10^8 kg, corresponding to a volume

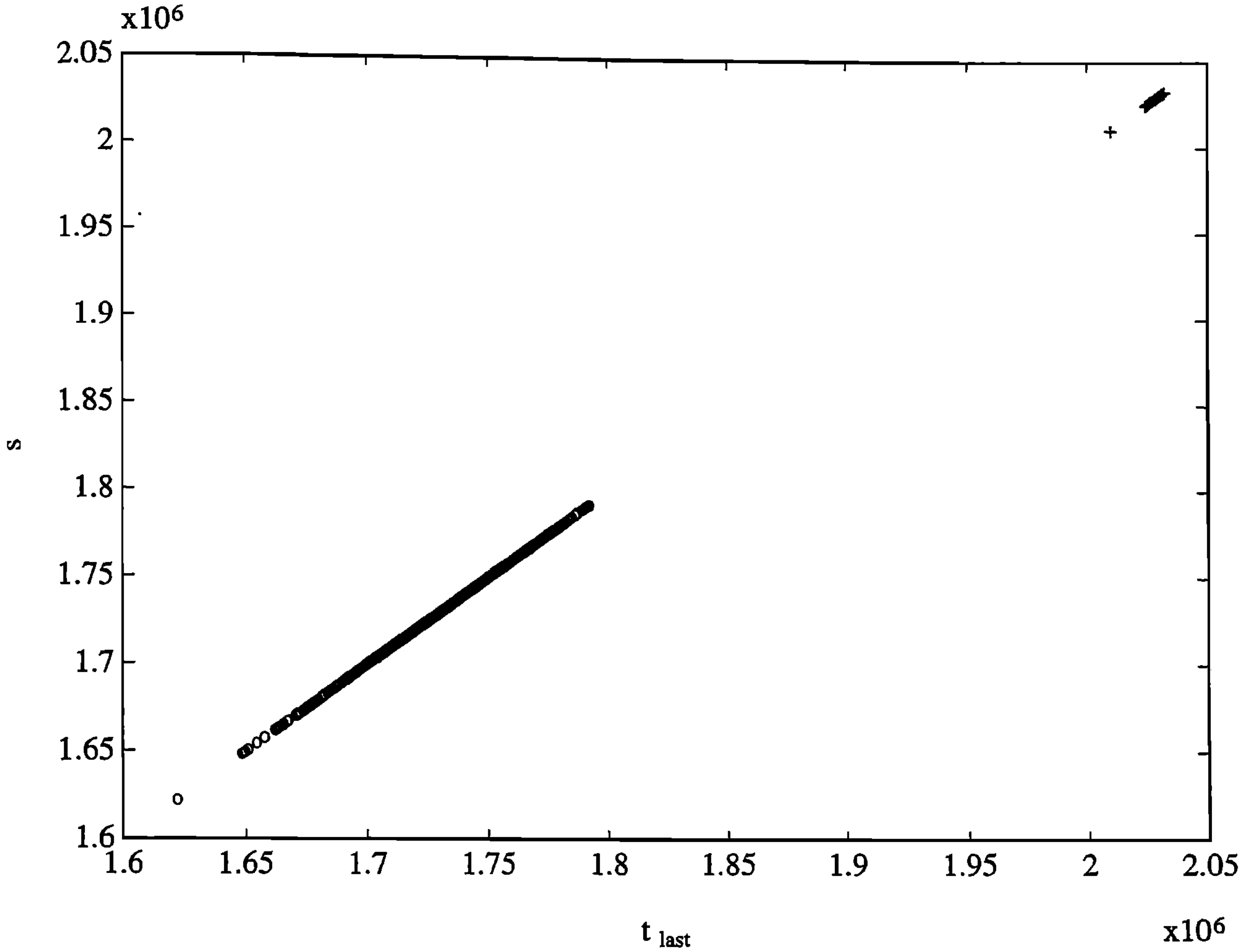


Figure 3. Time since last burst t_{last} (in units of Δt) versus the burst size s (in units of Δm). Note that we have expanded the scale. Data with $D_{eff} = 1.7 \times 10^{-6}$ and 1.7×10^{-3} m^2 s^{-1} are labeled by circles and crosses, respectively. The regularity of outburst and the strong correlation between t_{last} and s can be seen, as the standard deviation of the data is small.

of 0.17±0.05 km^3 at STP. This is consistent with all the known estimations in the 1986 event [*Kling et al.*, 1987; *Kusakabe et al.*, 1989; *Giggenbach*, 1990; *Tietze*, 1992].

Besides neglecting horizontal diffusion, the only problem with the choice of parameters in the above simulation is the large mass of a typical CO_2 quantum (which equals 160 kg). What happens if Δm is small? Since the saturation curve of CO_2 and hence the values of m_{sat} and m_{res} are fixed, the decrease in Δm implies an increase in $(m_{sat} - m_{res})/\Delta m$. To simulate the effect of a small CO$_2$ quantum, we increase the $(m_{sat} - m_{res})/\Delta m$ used in the simulation. Fortunately, the regularity of outbursts increases as Δm decreases, so our conclusions remain valid. The reason is a combinatorial (and entropic) one. Consider a system exhibiting regular outbursts with a typical time between successive outbursts equaling $n\Delta t$. At time $n\Delta t$ after an outburst, we have a high probability of finding at least one grid point (i, j) with $m_{ij} \geq m_{sat}$. Suppose we reduce Δm by a half in the new system. Then we have to reduce Δt by a half as well because the mass injection rate is fixed. We consider the distribution of the mass of CO_2 in the grid points in the new system at time $2n\Delta t$ after an outburst. Compared with the old system, we have doubled the amount of CO_2 mass quantum into which we can be randomly and uniformly introduced. Simple combinatorial analysis shows that the expected value of the maximum m_{ij} among the grid points is less than m_{sat}. Thus it takes a longer time to trigger an outburst when δm is halved. Besides, once an outburst occurs, the number of grid points with m_{ij} greater than m_{res} also increases. Thus the regularity of outbursts increases as the typical mass quantum of CO_2 decreases [*Chan et al.*, 1995b].

Discussion

We have examined the gradual gas injection and sudden degassing of crater lakes by means of a simple diffusion-driven equation. On the basis of our results, the degassing of Lake Nyos in 1986 began probably from the lake bottom, instead of a chemocline. Assuming a constant gas injection rate of 0.28 kg s^{-1} and using the CO_2 concentration profile measured just after the 1986 outburst taken by *Tietze* [1987], we estimate the recurrence time of CO_2 gas outburst from Lake Nyos to be 37±10 years.

When water near the lake bottom becomes unstable due to oversaturation of CO_2 gas, upward movement of water occurs by convection. Propagation of a disturbance can be as fast as 0.7 m s^{-1} and so a disturbance takes about 1 hour to propagate across Lake Nyos. We agree with the proposal by *Evans et al.* [1994] that degassing occurs in two stages. The initial nonviolent stage may take an hour to develop. Later, an explosive stage may be completed within a minute. Two stages of degassing result because the propagation time of a disturbance is a function sensitive to the size of CO_2 gas bubbles in the water.

The large-scale convection in the lake occurs in an orderly fashion, similar to the Bénard rolls. The area of gas emission on the lake surface can be small, less than 10% of the lake surface. As the rate of gas release increases, percolation transition occurs. That transition suppresses complete circulation of water and destroys the stable Bénard rolls. This leads to a quick end of gas release, consistent with the reports of some eyewitnesses.

We describe the statistics of outburst using a cellular automaton model. If the gas injection rate is constant, the size of gas outburst and the time between successive bursts are regular and correlated. The typical volume of CO_2 released is 0.17±0.05 km^3 at STP. The interval between successive outbursts is 37 years. The regularity of the outbursts suggests that the spatial distribution of gas injection and the finite size of each CO_2 mass quantum injected each time are not important for the outburst statistics.

We did not consider gas release by an external trigger, variable gas injection rate, or incomplete degassing. A variable injection rate can alter the interval between successive gas outbursts, though the size of each outburst is unaffected. Incomplete degassing affects both the time interval and the size of each outburst. The most important parameter when predicting the time of next gas outburst is the amount of CO_2 accumulated within the lake.

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