When Does Competition Lead to Efficient Investments?*

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Abstract

The paper studies agents' general or specific investment decisions under different ownership structures in a thin, decentralized market where each agent's decision affects the decisions and welfare of other agents mainly through indirect market linkages. It focuses on the roles of both competition and ownership. An investor is more likely to make specific investments as an employee than as an owner. "Excess competition among investors" makes efficient, specific investments more likely. Otherwise, inefficient, general investments and irrelevance of ownership are more likely to result. The problem in which the choice variable is investment level, instead of investment type, yields less contrasting results.

KEYWORDS: bargaining, incomplete contracts, market competition, ownership

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1 Introduction

Since the seminal papers by Grossman and Hart (1986) and Hart and Moore (1990),[1] a vast literature using the property rights approach has emerged to examine the effect of asset ownership on the firm's investment decisions.[2] The present paper differs from the other papers in two aspects. First, the focus here is on the firm's decision between a general or specific investment. A software company can write a generic human resource management software that works for all firms, or write a tailor-made one for a particular firm. Likewise, a young worker may contemplate earning an MBA (general investment) or earning an MSc in Financial Engineering (specific investment). A frequent decision as it is, the choice between a general or a specific investment is somewhat neglected in the literature.

Second, this paper studies one firm's investment decision together with the decisions of all other firms in the market. More specifically, we study a two-sided market with pairwise production and each pair involving one seller and one buyer. Suppose seller A and buyer B plan to work together. In this case, they do not need to cooperate or trade with seller C; nor will seller C's investment affect the value that A and B jointly produce. Nonetheless, seller A is concerned about seller C's investment because it serves as an outside option for buyer B, affecting seller A's ex post bargaining payoff as well as her investment incentive. Likewise, seller C's investment decision is affected by seller A's decision, not to mention the decisions by other parties. This is in contrast with the standard Grossman-Hart-Moore (hereafter, GHM) framework, in which the focus is usually is on seller A and buyer B, while their respective outside options are exogenously given. The strategic interaction between the two firms and their alternative trading partners is thus ignored in their framework.

In our two-sided market, we assume that only sellers make investments and the choices are between specific investments and general investments. The specific investment produces a high value when the buyer for whom the investment is made is involved, but it produces a zero value otherwise; the general investment produces a low value whichever buyer is involved. There are two types of assets: the seller's asset and the buyer's asset. A seller and a buyer

[1] For a nontechnical introduction to the approach, see Hart (1995).

[2] See, e.g., Chiu (1998), Dasgupta and Tao (2000), de Meza and Lockwood (1998a,b), and Rajan and Zingales (1998).

need to have access to both of the assets in order to produce. We assume the standard GHM setting otherwise. The asset ownership is given at the outset and remains unchanged in the exercise. In stage 1 of the game, sellers make observable but unverifiable investment choices. In stage 2, sellers and buyers bargain, followed by production and surplus division.

This paper highlights the importance of an individual's asset ownership and market environment in which that individual is situated in determining his/her investment decision. The basic insight of our main results is illustrated by the small market depicted in Figure 1. Each agent is referred to by a name, while his/her asset ownership is indicated in parentheses (a_s and a_b denote the seller's asset and the buyer's asset, respectively). First consider Sarah's investment problem. By making a specific investment, she restricts the set of potential buyers to a single agent – the buyer for whom the investment is made, say Bill; in the bilateral monopoly between them, the most she can get from ex post bargaining is one half of the production value. By making a general investment, she is ensured of a price nearly equal to its production value; the reason is that, since buyers outnumber sellers, there is excess demand for sellers as partners. Therefore, as long as the (net-of-cost) production value of a specific investment less than doubles that of a general investment, Sarah will choose the general investment. By a similar argument, foreseeing Sarah's decision, Sophie will also choose the general investment.

Sellers	Buyers
$\{$Sarah,a_s,$a_b\}$	Ben
$\{$Sophie,$a_s\}$	$\{$Bill,$a_b\}$
	$\{$Billy,$a_b\}$
Sue	$\{$Bob,a_s,$a_b\}$

Figure 1: A small market with three sellers and four buyers

Sue's situation is different, however. Hampered by her lack of any assets, Sue cannot exploit the advantage of excess competition on the buyers' side and can work only with Bob. Bob does not want to compete against the other buyers for partnership with Sarah or Sophie as there are already too many buyers competing for partnerships with them. This leads to a bilateral monopoly between Sue and Bob; whatever Sue's investment is, they will share the surplus equally. As long as the specific investment is the efficient investment, Sue will always choose it. While owning more assets de-motivates Sarah and Sophie, paradoxically, owning no assets at all motivates Sue to make the efficient choice.

Note that this adverse effect of asset ownership need not occur in another market environment. This again can be illustrated using Figure 1, with the modification that Billy and his asset are now absent from the scene so that there are equal numbers of sellers and buyers. In this case, absent the excess competition among buyers, Sarah and Sophie always find it optimal to make specific investments so long as these investments are efficient. In this case, asset ownership does not impair the investment incentive.

In this paper, we formalize the above ideas and show that they hold in more complicated market environments, where one challenging task is to define "market environment" or "excess competition" operationally. Our adverse effect of ownership generalizes related results in Chiu (1998) and de Meza and Lockwood (1998). These papers differ from GHM in the bargaining solution used (the outside option principle is used), but conform with it in the exogeneity of outside option. In the current paper, we differ from GHM also in how the outside option is determined. Through the endogenization of the outside option, we obtain new insight on the effect of market competition on the incentive role of ownership. To better relate to previous results, we also study a variant model in which the investment choice is the level of investment. We find that the adverse effect is not as likely as under the investment type problem, suggesting that the actual adverse effect of ownership is more serious than it is predicted by Chiu and de Meza and Lockwood.

Two remarks about our framework are in order. Firstly, our paper does not have anything to say about the initial distribution of assets, except to compare different initial allocations. This is in common with the existing literature except for the interesting paper by Gans (2005). One might think of a random initial allocation depending on chance and historical events outside the model and then compare ex post allocations for efficiency. Secondly, we assume that sellers and buyers do not contract before sellers choose investments. Sellers know all preferences and characteristics of all potential buyers prior to their investment decisions, and there is no market friction that prevents sellers from meeting and negotiating with the buyers for whom specific investments are made.[3]

[3]An example of this environment is the following: Suppose person A has as an asset a search engine software programme for rare books she developed for a school project. She can invest in tailoring this software to meet the needs of academic buyers interested in rare books or to meet the needs of bookstores that desire to know the kinds of books popular among

A few recent papers are related to our work. de Meza and Lockwood (1998b) study investment decisions in a general equilibrium framework that allows repeated production as well as births and deaths of agents. They assume that the agents are atomistic and non-pivotal. In our work, we study one-shot investments in a thin market with non-atomistic agents. Ramey and Watson (2001) study the effects of market frictions in a dynamic matching model with match-specific investments and effort choices to maintain the productivity of the assets. Unlike our model, they assume market friction, represented by the probability of being rematched, in determining which seller a buyer will be able to contact. In any match, the worst punishment for shirking is to sever the relationship. Thus, a frictionless market could have adverse effects on incentives. Moreover, Ramey and Watson do not study the problem of the *type* of investment as we do here.

Cole, Mailath and Postlewaite (2001) and Felli and Roberts (2000) investigate to what extent, in the absence of sophisticated contracts, the holdup problem between sellers and buyers can be mitigated through the market. The former paper focuses on the use of assortative matching, while the latter uses Bertrand competition. Unlike the present paper, neither of these papers analyzes the role of ownership in enhancing efficiency. Bolton and Xu (1999) construct a two-firm model to investigate the interaction between managerial market competition (both within and across firms) and ownership of firms. The firms in their model can consist of more than two agents, which allows the authors to compare the performance of a richer set of ownership structures. Grossman and Helpman (2002) construct a general equilibrium model where the market structure is endogenized and study the optimality of the equilibrium outcome. Atomistic and non-pivotal agents are assumed in their model. In all of these papers, the main focus is on the choice of investment level, instead of on the investment type.[4] Finally, in a sequel (Chatterjee and Chiu, 2006) to this paper, we address a different kind of market friction, one arising from commitments made as a result of the bargaining institution, and include choices on the flexibility of investment and the level of investment in characterizing the optimal ownership structure.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 introduces the concept of the efficient supply of general investments,

different categories of buyers. She knows both types of buyer are out there in the real world and she can go out and talk with them about her software, but she realizes that she cannot convince them that she has something useful until she undertakes the investment. In another example, in the labor market, individuals decide whether to earn an MBA (generalist) or an MS in computational finance (specialist) without pre-contracting with potential employers.

[4] See also Kranton and Minehart (2000)

allowing our bargaining solution and its justifications to be presented in Section 4. Section 5 studies the investment choice given the bargaining solution. Section 6 provides two extensions, one assuming noninvesting yet productive sellers, the other involving choices of investment levels. Section 7 offers some concluding remarks.

2 The Model

We consider a two-sided market with pairwise production, each pair involving one seller and one buyer. The set of sellers and the set of buyers are denoted by S and B, respectively. There are two types of assets: the seller's asset and the buyer's asset, denoted by a_s and a_b, respectively. They are important because each pair of seller and buyer needs to have access to both assets for the production to take place; we assume exclusive use of assets so that each single asset can only be used by a seller and buyer pair. Only the seller makes an investment – in her human capital – and the investment choice is between a general investment and a specific investment. Both types of investment require the same cost, $c > 0$. Provided that the required assets are available, a general investment yields a value of $m > 0$ whichever buyer is involved; a specific investment yields a value of $M > m > M/2$ when the buyer for whom the investment is made is involved, but a value of zero otherwise. To be economically interesting, we assume that $m/2 - c > 0$.

As is standard in the property rights literature, we assume that asset ownership is exogenously given and remains unchanged in the exercise. To facilitate the discussion, we partition the set of sellers, S, into three subsets: S^1, S^2, and S^3. S^1 consists of those sellers each of whom has a pair of a_s and a_b; S^2 consists of those sellers each of whom has a piece of a_s; and S^3 consists of those sellers who own no assets. Similarly, we partition the set of buyers, B, into three subsets: B^1, B^2, and B^3. B^1 consists of those buyers who own no assets; B^2 consists of those buyers each of whom has a piece of a_b; and B^3 consists of those buyers each of whom owns a pair of a_s and a_b. We do not consider any other ownership structures, such as a buyer owning one piece of a_s or a seller owning two pieces of a_s, etc. Players in subsets S^i and B^i are called type i players and their ownership structures are called type i ownership structures, where $i = 1, 2, 3$.

Given the ownership configuration, sellers and buyers engage in a two-stage game. In stage 1, sellers choose investments. By incurring a cost of $c > 0$, each seller can make either a general investment or a specific investment; investments are observable but not verifiable. In stage 2, sellers and buyers bargain on with whom to form a pair and at what price. As is standard in the property rights literature, bargaining at this stage is contractible, investments are non-resalable, and any agreement reached will be implemented accordingly. There is no discounting; the structure and payoffs of the game are commonly known.

Before we end this Section, it is important to note the varying power bestowed by asset ownership to agents in choosing their trading partners. Since a pair of a_s and a_b are required for a seller and a buyer to engage in production, a seller in S^i can only be matched with some buyer in B^j where $j \geq i$; in other words, a buyer in B^j finds investments by sellers useful only when the sellers come from S^i where $i \leq j$. The power of a seller in S^i is decreasing in i while the power of a buyer in B^i is increasing in i; here, power is defined as the ability to match to a larger set of partners. To illustrate this point, in Figure 1, Sarah – the seller in S^1 – can make an investment useful to Ben, Billy, or Bob; on the other hand, Sue – the seller in S^3 – can make an investment useful only to Bob, but not to Ben or Billy. This is the key to understanding the results of this paper.

3 Efficient Allocation and Excess Supply

The game is solved by backward induction. Before going into details about the bargaining process in stage 2, we first introduce some auxiliary notation to describe succinctly the sellers' investment choices in stage 1. We classify sellers in $S^i, i = 1, 2, 3$, into different groups depending on their investment choices. S^i_1 contains those in S^i who made a general investment. S^i_2 contains those who made a specific investment for some buyer in $B^l, l \geq i$ and for this buyer no other seller in $S^r, r \leq l$ has the same investment. S^i_3 contains those who made a specific investment for some buyer in $B^l, l \geq i$ while for this buyer some other seller in $S^r, r \leq l$ has made the same investment. The specific investments made by sellers in S^i_2 are nonduplicated, in contrast to those made by sellers in S^i_3, which are duplicated. Finally, we define S^i_0 to contain those sellers who made no investment at all and S^i_4 to contain those who made a specific investment but not specific to anybody in $B^l, l \geq i$. Clearly, sellers in S^i_0 and S^i_4 will play no role in the bargaining, either because they have made no investments at all or because no buyers exist who, together with the seller, have the required access to a_s and a_b for production. (Since members in S^i_0 and S^i_4 play no role in the bargaining and consequently get a zero payoff in the bargaining process, we do not explicitly specify their payoffs in the description of the bargaining solution.) Clearly, $S^i = \cup_{j=0}^4 S^i_j$ and $S^i_j \cap S^i_k = \phi$ for $j \neq k$.

Definition 1 *We call the investments made by sellers in* S_2^i, S_3^i, *and* S_4^i *the nonduplicated investments, the duplicated investments, and the useless investments, respectively, where* $i = 1, 2, 3$.

Similarly, we have a partition of buyers in $B^i, i = 1, 2, 3$. B_1^i contains those buyers in B^i for whom no seller in $S^r, r \leq i$ has made specific investments; B_2^i contains those buyers for whom there is only one seller in $\cup_{r \leq i} S^r$ who has made a specific investment; B_3^i contains those buyers for whom there are more than one seller in $\cup_{r \leq i} S^r$ who has made specific investments. Clearly, $B^i = \cup_{j=1}^3 B_j^i$ and $B_j^i \cap B_k^i = \phi$ for $j \neq k$.

There is an interesting property from the partitioning of S^i and $B^i, i = 1, 2, 3$: regardless of the investment choices in stage 1 of the game, there is a one-to-one onto mapping from $\cup_i S_2^i$ to $\cup_i B_2^i$ and we must have

$$\sum_{i=1}^{3} |S_2^i| = \sum_{i=1}^{3} |B_2^i|, \tag{1}$$

i.e., the number of sellers who have made nonduplicated investments equals the number of buyers for whom nonduplicated investments have been made.

3.1 Excess Supply of General Investment

In the property rights literature, the bargaining stage is always assumed to be efficient. (To ease the discussion, we may use "a seller selling her investment to a buyer" or "assigning a seller's investment to a buyer" to refer to the formation of a pair for production between the seller and the buyer.) In our context, given the investment decisions, an efficient allocation can be easily characterized here: For each nonduplicated investment, give it to the buyer for whom it is specific; for each kind of duplicated investments, give one unit to the buyer for whom it is specific; finally, the number of general investments being assigned should be maximized subject to the asset ownership constraints.

Despite possible multiplicity of efficient allocations, we now provide some characterization that is invariant for all efficient allocations given investment choices. We first consider a thought experiment in which investments are allocated efficiently subject to general investments being traded among each type of firms. Define $\Delta^i \equiv |S_1^i| - |B_1^i|$, which we call the constrained excess supply of general investment faced by type i firms, $i = 1, 2, 3$. The following must hold in this thought experiment. (i) In case $\Delta^i > 0$ (excess supply), while every buyer in B_1^i obtains a general investment, there are Δ^i sellers in S_1^i who are unable to have their general investments allocated; (ii) in case $\Delta^i = 0$

(market clearing), every buyer in B_1^i obtains a general investment and every seller in S_1^i has her general investment allocated; (iii) in case $\Delta^i < 0$ (excess demand), while every seller in S_1^i has her general investment allocated, there are $|\Delta^i|$ buyers in B_1^i who are unable to obtain a general investment. In other words, once we know Δ^i, we can say concretely what will happen to buyers and sellers in B_1^i and S_1^i.

Building on this, we now construct a general index E^i that answers similar questions when general investments are not restricted within the same type of firms. To fix the idea, consider the problem from the perspective of type 2 firms and suppose Δ^2 is positive. Once across-type allocation is allowed, this excess supply faced by type 2 firms may be, on the one hand, alleviated by a negative Δ^3 (since buyers in B_1^3 not receiving general investments from S_1^3 may want to acquire them from S_1^2) and, on the other hand, worsened by a positive Δ^1 (since sellers in S_1^1 unable to sell general investments to B_1^1 now compete with sellers in S_1^2). Taking into account the restrictions of feasible trade imposed by asset ownership, we come to conclude that, to evaluate type 2 firms' position fully, the correct index to use is

$$E^2 \equiv \max\{0, \Delta^1\} + \Delta^2 + \min\{0, \Delta^3\}, \tag{2}$$

which we call the (unconstrained) excess supply of general investments faced by type 2 firms.[5]

The index has interesting implications. If $E^2 = 0$ (market clearing), in any efficient allocation, every seller in S_1^2, as well as every more endowed seller in S_1^1, has her general investment allocated, while every buyer in B_1^2, as well as every more endowed buyer in B_1^3, is allocated a general investment. If $E^2 > 0$ (excess supply), buyers in B_1^2 (a fortiori, more endowed buyers in B_1^3) are in a favorable position. In any efficient allocation, every buyer in $B_1^2 \cup B_1^3$ is allocated a general investment, but some seller in $\cup_{j=1}^3 S_1^j$ must have her general investment unallocated. Moreover, there exists an efficient allocation in which there are exactly E^2 sellers in $S_1^1 \cup S_1^2$ with their general investments unallocated. If $E^2 < 0$ (excess demand), sellers in S_1^2 (a fortiori, more endowed

[5] The max{} and min{} reflect the varying power bestowed by asset ownership to agents in choosing their trading partners–the key insight in the analysis of the paper. Since sellers in S_1^3 are unable to compete with sellers in S_1^2 in selling to B_1^2, excess supply for general investment among type 3 firms ($\Delta^3 > 0$) will not worsen the position of sellers in S_1^2. This accounts for the use of $\min\{0, \Delta^3\}$ instead of Δ^3. Likewise, due to the lack of assets, buyers in B_1^1 will never be interested in acquiring general investments from sellers in S_1^2. Hence, an excess demand for general investments among type 1 firms ($\Delta^1 > 0$) will not improve the position experienced by sellers in S_1^2. This accounts for the use of $\max\{0, \Delta^1\}$ instead of Δ^1.

sellers in S_1^1) are in a favorable position. In this case, in any efficient allocation, every seller in $S_1^1 \cup S_1^2$ has her general investment allocated, while some buyer in $\cup_{j=1}^3 B_1^j$ is not allocated a general investment. Moreover, there exists an efficient allocation in which there are exactly $|E^2|$ buyers in $B_1^2 \cup B_1^3$ without general investments unallocated.

These implications are illustrated in the following examples, where a 6-tuple $[|S_1^1|, |S_1^2|, |S_1^3|; |B_1^1|, |B_1^2|, |B_1^3|]$ is used to represent the general investment choices made.

Example 1 *Consider an environment [2,1,0;0,1,1] such that $E^2 = 1 > 0$. Then, in any efficient allocation, both buyers in $B_1^2 \cup B_1^3$ are allocated the general investment, and exactly one seller in $S_1^1 \cup S_1^2$ has her investment unallocated.*

Example 2 *Consider an environment [1,1,0;2,1,1] such that $E^2 = -1 < 0$. Then in any efficient allocation, both sellers in $S_1^1 \cup S_1^2$ have their investment allocated, and some buyer in $\cup_{j=1}^3 B_1^j$ is not allocated a general investment. Moreover, in those efficient allocations in which the investment from S_1^1 is assigned to B_1^1, one buyer in $B_1^2 \cup B_1^3$ is not allocated a general investment.*[6]

In general, we can define $E^i, i = 1, 2, 3$ to measure the excess supply of general investments faced by type i firms. Essentially, it is the sum $\Delta^1 + \Delta^2 + \Delta^3$ with modification, taking into account the effect of asset ownership in constraining the choice of trading partners. Specifically, we define

$$E^1 \equiv \Delta^1 + \min\{0, \Delta^2 + \min\{0, \Delta^3\}\}, \tag{3}$$
$$\text{and} \quad E^3 \equiv \max\{0, \max\{0, \Delta^1\} + \Delta^2\} + \Delta^3 \tag{4}$$

as the excess supply of general investments faced by type i firms, $i = 1, 3$. (The nested min{} and max{} again reflect constraints from asset ownership on agents in their choice of trading partners.) We establish the following general result (all proofs of lemma and propositions, unless otherwise stated, are relegated to the Appendix):

[6]Note that no buyer in $B_1^2 \cup B_1^3$ is unallocated in the efficient allocation in which the investment from S_1^1 is given to B_1^3 and that from S_1^2 is given to B_1^2. Thus, it is erroneous to claim that, in any efficient allocation, somebody in $B_1^2 \cup B_1^3$ must be unallocated.

Lemma 1 *Given any market structure and given any investment decisions. The following are true, for $i = 1, 2, 3$:*

1. *(Excess supply) Suppose $E^i > 0$. For all efficient allocations, some seller in $\cup_{j=1}^{3} S_1^j$ has her general investment unallocated and every buyer in $\cup_{j \geq i} B_1^j$ is allocated a general investment. In addition, there exists an efficient allocation in which the number of sellers in $\cup_{j \leq i} S_1^j$ with their general investments unallocated is exactly equal to E^2.*

2. *(Market clearing) Suppose $E^i = 0$. For all efficient allocations, every seller in $\cup_{j \leq i} S_1^j$ must have her general investment allocated and every buyer in $\cup_{j \geq i} B_1^j$ is allocated a general investment.*

3. *(Excess demand) Suppose $E^i < 0$. For all efficient allocations, every seller in $\cup_{j \leq i} S_1^j$ has her general investment allocated and some buyer in $\cup_{j=1}^{3} B_1^j$ is not allocated with a general investment. There exists an efficient allocation in which the number of buyers in $\cup_{j \geq i} B_1^j$ without their general investments allocated is exactly equal to $|E^2|$.*

A remark is in order here. Despite multiple feasible allocations, if we insist that allocations be in the core, then all efficient allocations must be payoff equivalent. For instance, if $E^2 > 0$ such that some seller in $S_1^1 \cup S_1^2$ has her general investment unallocated and receives a zero payment in one efficient allocation, then that seller must also receive the same zero payment in all efficient allocations. Core allocation also implies that any seller equally or less endowed must also receive zero payments in all efficient allocations.

4 Bargaining

Our bargaining solution consists of two components: that the allocation of investments is efficient, and that the transfers satisfy the following scheme, for $i = 1, 2, 3$:

1. Players who neither give out nor receive any investment receive a zero price.

2. Buyers in B_3^i who receive a duplicated investment pay the same price, $p_3^i = 0$, to the respective sellers.

3. Buyers in B_1^i who receive a general investment pay the same price, p_1^i, to the respective sellers.

4. Buyers in B_2^i who receive a nonduplicated investment pay the same price, p_2^i, to the respective sellers.

5. p_1^i and p_2^i are given in the following table.

E^i	p_1^i	p_2^i
$=0$	$m/2$	$M/2$
<0	m	$M/2$
>0	0	$M-m$

Table 1: The price table

Properties 1 to 3 are requirements of the outcome being in the core: any violation of them will move the outcome away from the core. Property 4 is a symmetry property that we find appealing. Property 5 deserves more explanation. According to the table, the price of general investments will be driven up to m or down to zero to clear the market in case of excess demand or excess supply. This is simply an implication of the core allocation. In the case of market clearing, the ex post surplus of a general investment, m, is equally split between the seller and the buyer. This can be seen as a generalization of the Nash bargaining outcome (Nash 1950; Rubinstein 1982).

For nonduplicated investments, the prices follow the outside option principle in noncooperative bargaining theory: The buyer will obtain half of the surplus from bargaining when his outside option is nonbinding and will obtain a value exactly equal to his outside option otherwise. In case $E^i = 0$ or $E^i < 0$, the price of a general investment is $m/2$ or m, respectively. For a buyer in B_2^i, acquiring a general investment (his outside option) brings him a gain of $m/2$ or zero, which is strictly less than an equal split of the surplus from the nonduplicated investment, M. In this case, the outside option is nonbinding and the price at which the nonduplicated investment is traded is simply $M/2$. On the other hand, in the case of $E^i > 0$, the price of a general investment is zero. Acquiring a general investment (his outside option) brings the buyer a gain of $m > M/2$. In this case, the outside option is binding. To maintain the buyer's gain of m, the price at which the nonduplicated investment is traded is simply $M - m$.[7]

Remark 1 *It is routine to check that $E^1 \leq E^2 \leq E^3$ for all Δ^1, Δ^2, and Δ^3. This implies that $p_1^1 \geq p_1^2 \geq p_1^3$ for all Δ^1, Δ^2, and Δ^3.*

[7] See Binmore, Rubinstein, and Wolinsky (1986) and Sutton (1986) for the outside option principle: and see Binmore, Shaked, and Sutton (1989) for experimental support of the principle.

Remark 1 indicates how asset ownership empowers sellers to match with a large group of potential buyers: sellers in S^i can only sell their investments to B^j, where $j \geq i$. Hence, the general investments from type 1 sellers cannot be cheaper than those from type 2 sellers; in turn, those general investments from type 2 sellers cannot be cheaper than those from type 3 sellers.

4.1 Justifications for the Bargaining Solution

We have proposed to use a bargaining solution characterized by efficient allocation together with the five properties regarding prices. Note that since our bargaining solution prescribes that all players on the long side receive a zero payoff, this rules out collusion among sellers or among buyers. This is justifiable because any such collusion is not immune to deviation. In fact, any allocation that is in the core must have the property that players on the long side receive a zero payoff. The following Lemma is easy to prove.

Lemma 2 *1. The allocation resulting from our bargaining solution is in the core.*

2. If an allocation is in the core, then the following must hold true:

(a) *efficient matching, i.e., matching is such that social welfare is maximized.*

(b) *for* $E^i > 0, p_1^i = 0, i = 1, 2, 3.$

(c) *for* $E^i < 0, p_1^i = m, i = 1, 2, 3.$

(d) $p_3^i = 0, i = 1, 2, 3.$

(e) $p_1^i = 0 \Rightarrow p_2^i \leq M - m.$

Two less controversial properties are used to arrive at our bargaining solution: namely (i) the surplus is split equally when the outside option is nonbinding, and (ii) the payoff a player receives from a bilateral relationship simply matches his/her outside option if the outside option is binding; these two properties can also be summarized as Nash equilibrium with outside options as constraints. Since requiring that the bargaining solution be in the core is fairly legitimate, we think the bargaining solution reasonable.

Our bargaining solution is a limiting equilibrium in a generalized alternating bargaining game a la Rubinstein and Chatterjee and Dutta (1998). Stage 2 of our game – the bargaining stage – consists of an infinite number of substages in which sellers and buyers alternate in making proposals.[8] Those responders who accept offers, together with the respective proposers, will disappear from the bargaining game. The remaining players will proceed to the next substage with their roles of proposers and responders switched. The process continues until all potentially beneficial trades are exhausted. The proof that this game yields our bargaining solution as a limiting outcome is relegated to the Appendix.

5 Investment Decisions

We now study the investment game using the bargaining solution previously stipulated. We first note the following result (proof omitted):

Lemma 3 *In any equilibrium, (1) $|S_3^i| = |S_4^i| = |B_3^i| = 0$ for all i; (2) $E^i \leq 0$ for all i.*

Part (1) is straightforward: the seller would not choose to invest duplicated investments or useless investments; otherwise, she would not be able to recoup the investment cost. Part (2) says that the case when excess supply of general investment will not result; otherwise, some seller with a general investment must receive a zero price and will be unable to recoup the investment cost.

In the first two subsections that follow, we make an additional assumption that $|S^i| = |B^i|$ for $i = 1, 3$. Simplifying as it is, this assumption has a natural interpretation. Each pair of type 1 seller and type 1 buyer can be construed as an integrated firm with the seller as the employer and the buyer as the employee; each pair of type 3 seller and type 2 buyer can be construed as an integrated firm with the buyer as the employer and the seller as the employee. This assumption has an interesting implication: the difference in the number of sellers and the number of buyers is reflected by the difference in the number of type 2 sellers and buyers: $|S| - |B| = |S^2| - |B^2|$.

Given the additional assumption, the investment decisions depend on the ownership and competition in a neat way.

[8] We assume that each offer is publicly announced and available to any responder at the same, nonpreferential price. This restriction is made to simplify the exposition, but there is no particular reason why offers cannot have other properties. We conjecture that the equilibrium outcome that we are interested in survives the relaxation that private offers are allowed.

5.1 Excess Competition among Sellers ($|S| \geq |B|$)

In this subsection, we look at the scenario in which the number of sellers is greater than or equal to the number of buyers, $|S| \geq |B|$. The following proposition is easy to show (proof omitted).

Proposition 1 *Suppose that $|S| \geq |B|$. The following prescribes an equilibrium outcome:*

1. *All type 1 sellers (sellers in S^1) make nonduplicated investments specific for some type 1 buyer.*
2. *A total of $|B^2|$ type 2 sellers (sellers in S^2) make nonduplicated investments specific for some type 2 buyer.*
3. *All type 3 sellers (sellers in S^3) make nonduplicated investments specific for some type 3 buyer.*

In this equilibrium, ownership does not play any role in affecting the seller's investment decision; regardless of her asset ownership, the seller always chooses a specific investment whenever investing. The intuition is that there is strong enough competition among sellers in relation to buyers to force sellers to make specific investments when investing.

It is easy to check that the above prescription constitutes an equilibrium. Given their investment decisions, those sellers who made a general investment can sell at $m/2$ and those who made a nonduplicated investment can sell at $M/2$. If some seller $s^j \in S^j$ prescribed to make a nonduplicated investment unilaterally deviates to make a general investment, then both $|S_1^j|$ and $|B_1^j|$ increase by one without changing the fact that $E^i = 0$ for all i. As a consequence, the deviating seller receives just $m/2$ from bargaining, which is less than what she would receive under no deviation. Other deviations such as no investment at all, duplicated investment, or specific investment are clearly unattractive.

For any seller in S^2 prescribed not to make an investment, unilaterally deviating to choose a general investment leads to excess supply ($E^2 = E^3 = 1$) such that she will get a zero price from her investment and will be unable to recoup the investment cost. If the seller chooses specific investment instead, the investment must be duplicated and unprofitable. Therefore, the prescription in the Proposition must constitute an equilibrium.

While Proposition 1 describes one equilibrium, we next argue that the main message of the Proposition that only specific investments are chosen and ownership does not play a role is in fact invariant for all equilibria of the game.

Proposition 2 *Suppose that $|S| \geq |B|$. In every equilibrium outcome of the investment game, only $|B|$ nonduplicated specific investments are made and no general investments are made.*

The crucial thing to notice is that there are more sellers than buyers. Under any (pure strategy) equilibrium, there must be just $|B|$ sellers who would invest. Therefore, in the bargaining stage, the number of sellers who have invested is equal to the number of buyers who want to buy. This leads to an equal split of the surplus and the benefit of investing a general investment is thus lower than that of a specific investment, so long as the seller has decided to invest. Technically speaking, in all equilibria, we have $E^i = 0$ and $\Delta^i = 0$ for all i.

The proposition also points out two kinds of indeterminacy in the game. In the first kind, one seller makes and sells her specific investment to a different buyer in the two equilibria. This indeterminacy is payoff irrelevant because each party's payoff does not vary across two equilibria. In the second kind, the number of independent sellers that make and sell specific investments is different from one equilibrium to the other, as is the number of type 3 sellers that make and sell specific investments. Therefore, when all equilibria are likely ex ante, absence of ownership renders the seller owning fewer assets to have a smaller probability of trade.

5.2 Excess Competition among Buyers ($|S| < |B|$)

We now study the opposite case when $|S| < |B|$. In contrast to the previous case, ownership matters but in a way different from Hart and Moore's general insight which argues that assigning more assets to an agent gives the agent a more appropriate incentive to invest.

Proposition 3 *Suppose that $|S| < |B|$. Every equilibrium of the game has the following properties:*

1. *All type 1 sellers (sellers in S^1) make a general investment.*
2. *All type 2 sellers (sellers in S^2) make a general investment.*
3. *All type 3 sellers (sellers in S^3) make a nonduplicated investment (specific for some type 3 buyer (buyer in B^3)).*

In this scenario, only type 3 sellers – investors with the fewest assets – will have the appropriate incentive to choose specific investments; all other sellers choose the inefficient general investment. This result holds true for all equilibria of the game. The Proposition can be illustrated by Figure 1. In equilibrium, both Sarah and Sophie make general investments for m while Sue makes a specific investment for Bob for $M/2$. Sarah and Sophie can extract all the surpluses from their general investments because three buyers–Ben, Bill and Billy–compete for their general investments. Sue cannot emulate Sarah and Sophie because of her lack of assets. She can only trade her investment with Bob, and making a specific investment for Bob is therefore the best choice for her.

Consider a general problem in which $|S| < |B|$. Regardless of the sellers' investments, there must be buyers in $B^1 \cup B^2$ who end up not having any investment. As a consequence, the equilibrium price of a general investment will be driven up to m, while the equilibrium price of a nonduplicated specific investment is still $M/2$, reflecting the bilateral monopoly between the seller and the buyer. Since any seller $s^1 \in S^1$ is eligible to sell to any buyer in $B^1 \cup B^2$, she is guaranteed a price of m for a general investment and a price of $M/2$ at most for a specific investment. To choose the general investment is indeed s^1's dominant strategy. Given that all sellers in S^1 choose the general investment, the same logic argues that all sellers in S^2 also have the general investment as their dominant strategy. The excess demand is the crucial reason for why both S^1 and S^2 make general investments in any equilibrium here, but do not in Propositions 1 and 2 with $|B| \leq |S|$. The general lesson found in Hart and Moore that asset ownership enhances the asset owner's incentive to make more appropriate investments does not hold in this context.

The above analysis points out that the driving force for the inefficiency in the market is that sellers take advantage of the excess competition among buyers. To restore efficiency, sellers should refrain from taking advantage of the competition. Type 3 sellers have to refrain from doing so because, lacking any assets, they can bargain and trade only with buyers in B^3 and not with buyers in B^1 or B^2. The excess competition among buyers is therefore eliminated.

5.3 A Generalized Framework

Thus far, we have restricted our attention to the case when $|S^i| = |B^i|$ for $i = 1, 3$.[9] We now plan to relax this assumption. Figure 2 depicts a market in which there are one type 1 seller (Sarah) and two type 1 buyers (Ben and Bill).

[9] We still make the following restrictions: (1) each player cannot own more than one a_s and one a_b; (2) a seller cannot own one unit of a_b only; and (3) a buyer cannot own one unit of a_s only.

Type of Firms	Sellers	Buyers
1st	{Sarah,a_s,a_b}	Ben
1st		Bill
2nd	{Sophie,a_s}	{Billy,a_b}
2nd	{Stephanie,a_s}	
3rd	Sue	{Bob,a_s,a_b}

Figure 2: A small market in which $|S^1| \neq |B^1|$

Once the assumption of $|S^i| = |B^i|$ for $i = 1, 3$ is relaxed, it is not that easy to characterize the market by a single number or a single relationship (such as whether or not $|S| \geq |B|$) of which the equilibrium investment pattern can be made a function. It turns out that a notion parallel that of excess supply of general investments is very useful in summarizing the new results here. Define

$$\begin{aligned} e^1 &\equiv d^1 + \min\{0, d^2 + \min\{0, d^3\}\} \\ e^2 &\equiv \max\{0, d^1\} + d^2 + \min\{0, d^3\} \\ \text{and} \quad e^3 &\equiv \max\{0, \max\{0, d^1\} + d^2\} + d^3, \end{aligned}$$

where $d^i = |S^i| - |B^i|, i = 1, 2, 3.$

The interpretation of these terms is very similar to that of E^i, despite the fact that the former are pre-investment market characteristics while the latter are post-investment ones. Essentially, e^i indicates whether or not there are more sellers competing with a seller in S^i than buyers competing with a buyer in B^i when taking asset ownership into account. When $e^i > 0$ ($e^i < 0$), there are more (fewer) sellers competing with a type i seller than buyers competing with a type i buyer; when $e^i = 0$, the number of sellers competing with a type i seller equals the number of buyers competing with a type i buyer. As an illustration, consider the following situation: $\Delta^1 = 4$, $\Delta^2 = 2$, and $\Delta^3 = -3$ and, accordingly, $e^2 > 3$. In this case, two sellers in S^2 have to compete with four sellers in S^1 to sell to three buyers in B^3. Since investments that can be successfully sold cannot outnumber buyers, some seller from S^2 or her rival from $S^i, i \neq 2$, is bound to have no trade. The predicament faced by "some seller in S_1^2" is not limited to a specific seller—it is perceived equally by every seller in S_1^2. On the other hand, buyers in B_1^2 are in so favorable a position that each is certain to get an investment in equilibrium.[10]

[10] Like the case of E^i, the max{.} and min{.} simply reflect constraints imposed on agents by asset ownership on their trading opportunities. Consider the case $d^1 = 4$, $d^2 = 2$, and

We present the following results that generalize the previous results.

Proposition 4 *1. In any equilibrium sellers of the same ownership type must have the same type of investment (specific versus general), as long as they invest.*

2. If some seller in S^i makes a specific investment, then any seller in S^{i+1} who invests must make a specific investment too, where $i = 1, 2$.

3. If $e^i \geq 0$, every investing seller in S^i makes a nonduplicated specific investment, $i = 1, 2, 3$.

4. If $e^i < 0$, every investing seller in S^i makes a general investment, $i = 1, 2, 3$.

There are two points worth noting here. First, sellers without any assets are more likely to make specific investments than are sellers with a_s, who in turn are more likely to make specific investments than are sellers with both types of assets. The basic insight on the adverse effect of ownership highlighted in the last section continues to hold in the more general setting. In fact, the first two results of the above Proposition mean that only the following four possibilities of investment choices (as long as sellers invest) are feasible equilibrium outcomes (Table 2):

outcome	S^1	S^2	S^3
(i)	general	general	general
(ii)	general	general	specific
(iii)	general	specific	specific
(iv)	specific	specific	specific

Table 2: Equilibrium investment decisions

Second, the notion of excess competition is extended and is captured by an index, $e^i, i = 1, 2, 3$. It is straightforward to show that $e^1 \leq e^2 \leq e^3$, which is

$d^3 = 0$, in which two extra sellers from S_1^2 need buyers. Since the buyers in B_1^1 are unable to utilize investments supplied by S^2, a negative d^1 will be treated as if it were a zero d^1 in determining e^2. Although some four buyers in B^1 are bound to receive no investments from S^1, they are unable to utilize those from S^2, hence not mitigating S^2's excess supply problem.

reminiscent of Remark 1 that says that $E^1 \leq E^2 \leq E^3$. Result 3 of Proposition 4 states that, in case of $e^1 > 0$ (excess supply), type 1 sellers will take advantage of the excess supply and will make general investments. Sellers with fewer assets may make general or specific investments with the restrictions imposed by Results 1 and 2. The admissible equilibrium outcomes in this case are outcomes (i) to (iii), but not (iv). Numerical examples exhibiting these three types of outcomes can be constructed. In the absence of excess competition on the buyers' side, according to Result 4 of the proposition, there must be some seller making a specific investment. In this case, the admissible equilibrium outcomes are outcomes (ii) to (iv), but not (i). Numeric examples exhibiting these three types of outcomes can be constructed. These insights regarding the roles of competition and ownership are consistent with those found in the previous section.

We end this section with two remarks. Firstly, since inefficient decisions are made because there is too little competition among sellers, one way to correct the incentive to allow easy entry into the sellers' market. By allowing enough sellers to enter the market, sellers would switch from making general investments to making specific investments. Secondly, since a reduction in the number of agents on either side of the market will increase the bargaining power and affect the sellers' optimal investments, our results have strong implications for horizontal integration. A merger among a group of sellers, say, may trigger a marketwide change in investment decisions. Not only will sellers in the merged firm change their investment decisions, other sellers will have also change their investment choices.

6 Extensions

We now look at two variants of the model in order to help sharpen our understanding. The first variant is one in which a seller is still productive even if she does not make any investment; the second variant is one in which each seller chooses the investment level, rather than the investment type. For both variants, we return to the assumption of $|S^1| = |B^1|$ and $|S^3| = |B^3|$. To simplify the exposition, we focus on the cases of $|S| > |B|$ and $|S| < |B|$, while ignoring the specific case of $|S| = |B|$. Using the two properties of the bargaining solution that we have made use of—allocation being in the core and a Nash equilibrium with outside options as constraints, we are able to identify in these two extensions equilibria that resemble those found in earlier Sections.

6.1 Noninvesting yet Productive Sellers

Let the surplus created by a noninvesting seller and any buyer be the same and equal to k. The economically interesting case is when $k < m - c$ (in the case when $k \geq m - c$, no seller could ever consider a general investment optimal). Then it is natural to assume that the surplus jointly created by a buyer and a seller with a specific investment—but not specific to that buyer—is also k.We obtain the following properties (the proofs for the following four Propositions are omitted).

Proposition 5 *Suppose $|S| > |B|$. The following is an equilibrium. $|B_2|$ type 2 sellers and all type 3 sellers each make a nonduplicated investment for a type 2 and type 3 buyer, respectively, and receive a price of $\min\{M/2, M - k\}$. All type 1 sellers each make a nonduplicated investment for a type 1 buyer and receive a price of $M/2$ if $k/2 \leq M/2 - c$ and make no investment and receive a price of $k/2$ otherwise.*

To see this is an equilibrium, first note that, because of core allocation, any non-investing type 2 seller is willing to sell its zero investment at a price of zero, giving a surplus of k to the buyer so long as he has the adequate asset(s). This outside option of k is available for type 2 and type 3 buyers, but not type 1 buyers. Therefore, using Nash equilibrium with outside options as constraints, we show that type 2 and type 3 buyers receive a price of $\max\{M/2, k\}$ and type 1 sellers receive a price of $M/2$ or $k/2$, depending on the type 1 sellers' investments. The payoffs to sellers as described in the proposition are then verified.

Next, we show that the prescribed investments indeed constitute optimal responses among sellers. Consider a type 2 or type 3 seller who is prescribed to invest. By unilaterally deviating to a general investment, due to Nash equilibrium with outside options as constraints, she obtains $\min\{m/2, m - k\} - c$, which is lower than her equilibrium payoff of $\min\{M/2, M - k\} - c$. By not investing, since the allocation is in the core, she has a payoff of 0, which is also lower and undesirable. Now we consider the payoff of a type 1 seller. By choosing a specific investment, a general investment, or no investment, according to Nash bargaining with outside options as constraints, she obtains a payoff of $M/2 - c$, $m/2 - c$, or $k/2$, respectively. Hence, a nonduplicated investment is best when $k/2 \leq M/2 - c$ and no investment is best otherwise. Finally, it suffices to note that those prescribed not to invest will not gain by investing.

Proposition 6 *Suppose $|S| < |B|$. The following is an equilibrium. All type 1 sellers and type 2 sellers each make a general investment and receive a price of*

m. All type 3 sellers each make a nonduplicated investment for a type 3 buyer and receive a price of $M/2$ if $k/2 \leq M/2 - c$ and each make no investment and receive a price of $k/2$ otherwise.

Given the investments as prescribed, the prices are obtained according to the allocation being in the core and Nash bargaining with outside option as constraint. By making the general investment, type 1 and type 2 sellers obtain all the surplus of their general investments through exploiting the excess competition among type 1 and type 2 buyers. This gives them an even higher payoff than making a specific investment. Type 3 sellers cannot make use of the excess competition among buyers, and hence choose the best decision foreseeing an equal division of surplus. Their optimal decisions are to make specific investments when $k/2 \leq M/2 - c$ but no investment otherwise.

To summarize, when k is small, the predicted investment pattern is just the same as the case when $k = 0$. When k is sufficiently large ($k/2 \leq M/2 - c$), the results are somewhat different, strengthening our results when $|S| > |B|$ and weakening them when $|S| < |B|$. We view this case of sufficiently large k to be rather specific and of limited interest.

6.2 Sellers Choosing Investment Levels

This second extension considers the scenario in which the choice is on the investment level. In other words, each seller's decisions are on the level of a specific investment and for whom the investment is specific. Denote the investment level as e, and the corresponding cost as $c(e)$, which is increasing, strictly convex, twice differentiable, and satisfies $c(0) = 0$ and $c'(0) = 0$. The surplus that the seller creates together with a buyer (along with a pair of a_s and a_b) is $f(e)$ if the buyer is the one for whom the investment is specific, and it is $g(e)$ otherwise. The two functions, $f(e)$ and $g(e)$, have the following properties: $f(0) = g(0) = k \geq 0$, $f(e) > g(e)$ and $f'(e) > g'(e)$ for $e > 0$. Define $e_{FB} \equiv \arg\max_e\{f(e) - c(e)\}$, $e^* \equiv \arg\max_e\{f(e)/2 - c(e)\}$, $\widetilde{e} = \arg\max_e \{\min\{f(e)/2, f(e) - k\} - c(e)\}$, and $\overline{e} = \arg\max_e\{\max\{f(e)/2, g(e)\} - c(e)\}$. These investment levels, which we assume are unique, have the following interpretations. e_{FB} is the first-best investment level; e^* is the seller's optimal investment level foreseeing equal division of the surplus; $\widetilde{e}$ is the seller's optimal investment level foreseeing that the targeted buyer has an outside option of k; and $\overline{e}$ is the seller's optimal investment level foreseeing that she could sell her investment to a non-targeted buyer at $g(e)$, where e is her chosen investment level.

Proposition 7 *Suppose* $|S| > |B|$*. The following is an equilibrium. All type 1 sellers makes an investment* e^* *for some type 1 buyer and receive a payoff of* $f(e^*)/2 - c(e^*)$*.* $|B_2|$ *type 2 sellers and all type 3 sellers each make an investment* $\widetilde{e}$ *for a type 2 and a type 3 buyer respectively and receive a payoff of* $\min\{f(\widetilde{e})/2, f(\widetilde{e}) - k\} - c(\widetilde{e})$*.*

Given the prescribed investments, using the core allocation and Nash bargaining with outside options as constraints, one can easily verify the prices as described. To show the Proposition, it suffices to show that no seller can benefit from a unilateral deviation in investments. Choosing a different level of investment is not optimal, since the prescribed levels, e^* and $\widetilde{e}$, by definition, are those that maximize, respectively, type 1's seller's payoff $f(e)/2 - c(e)$ and type 2 or type 3 seller's payoff $\min\{f(e)/2, f(e) - k\} - c(e)$. Investing for a different buyer is undesirable as it leads to duplication. Choosing no investment at all is undesirable too: for type 1 sellers, it means a payoff of $k/2 = f(0)/2 - c(0) < f(e^*)/2 - c(e^*)$ (by definition of e^*); for type 2 or type 3 sellers, it means a payoff of zero.

It is interesting to note the relationship between $\widetilde{e}$ and e^*. It is straightforward to verify that $\widetilde{e} = e^*$ when $k \leq f(e^*)/2$ and $e_{FB} \geq \widetilde{e} > e^*$ otherwise. In other words, when k is small enough, ownership has no effect on investment and is irrelevant. When k is sufficiently large, on the other hand, sellers with fewer assets now have a stronger incentive to invest. This illustrates the insight in Chiu and De Meza and Lockwood (1998a) and is consistent with our results found for the investment type problem.

Proposition 8 *Suppose* $|S| < |B|$*. The following is an equilibrium. All type 1 and type 2 sellers makes an investment* $\overline{e}$ *for some type 1 and type 2 buyer, respectively, and receive a price of* $\max\{f(\overline{e})/2, g(\overline{e})\}$*. All type 3 sellers each make an investment* e^* *for a type 3 buyer and price a payoff of* $f(e^*)/2$*.*

When $|S| < |B|$ and buyers are on the long side, type 1 and type 2 sellers take into account of their outside options with alternative buyers when investing. Their optimal investment level is given by $\max\{\max\{f(e)/2, g(e)\} - c(e)\}$ while that of type 2 sellers, who do not have outside options, is given by $\max f\{(e)/2 - c(e)\}$. Since $\overline{e}$ and e^*, by construction, are the optimal choices for the two problems, the proposition is straightforward and correct.

While the general insight of the proposition is similar to our earlier result in Section 4, the exact implication regarding efficiency is less conclusive. Depending on the relationship between $f(e)$ and $g(e)$, the more endowed type 1 and type 2 sellers may invest more efficiently than their type 3 counterparts.

In other words, asset ownership may be efficiency enhancing. To see this, suppose that $f(e) = k+e$ and $g(e) = k+\alpha e$, where $-\infty < \alpha < 1$.[11] For $0 < \alpha < 1$, an investment increases not only the surplus with the targeted buyer but also the surplus from alternative relationships. For $\alpha < 0$, the investment is so specific that it reduces the surplus in any alternative relationship.

By using the following specifications, (i) $k = 0$ and $\alpha > 1/2$; (ii) $k > 0$, $0 < \alpha < 1/2$, and $g(e^*) > f(e^*)/2$; and (iii) $k > 0$, $\alpha < 0$ but $k < f(e^*)/2 - c(e^*)$, we can easily show that (i) $e_{FB} > \overline{e} > e^*$, (ii) $\overline{e} < e^*$, and (iii) $\overline{e} = e^*$, respectively. Of particular interest to us is the first specification, which suggests a positive role of ownership and is consistent with the GHM framework. It is important to note that the underlying force for this investment-enhancing effect of ownership is exactly the same for the investment-weakening effect in the case of the investment type problem. In both problems, more endowed sellers attempt to take advantage of the excess competition on the part of buyers. Here, since $\alpha > 0$ and this outside option is greater than the equal division of surplus that will result from Nash bargaining without an outside option, the sellers invest a greater amount. In the investment type problem, the sellers choose the inefficient general investment in order to enjoy the excess competition. This thus suggests the crucial difference between the two kinds of investment problems.

Although the second specification does show one main thrust of the paper— the adverse effect of ownership— the effect here does not seem to be as convincing as under the investment type problem. After all, the result here relies on the property that, in the absence of any investment, not only the seller still produces a positive surplus but the surplus is also sufficiently large. Interesting as it may be, this property is a bit unusual and nonstandard.

The third specification, which means that a seller's outside option decreases with her own investment, is one of the formulations that Rajan and Zingales used in arguing for an adverse effect of ownership. Our result does not give the adverse effect of ownership as in Rajan and Zingales because a different bargaining solution is used. Under the Shapley value, an agent's outside option always enhances one's payoff from bargaining; under the outside option principle, an agent's outside option does not enhance one's payoff from bargaining until reaching a critical level. Hence, with the standard assumptions in Hart and Moore, where the outside option increases with investment, the Shapley value predicts a *more positive* role to asset ownership in enhancing

[11] The assumption of linearity of the payoff function in the investment is quite standard and is without loss of generality; see, for example, Tirole (1999) and Aghion and Tirole (1997) for similar formulations.

investment efficiency than the outside option principle does; but under Rajan and Zingales, where the outside option decreases with investment, it predicts a *more negative* role to asset ownership.

7 Concluding Remarks

This paper has demonstrated the joint effects of ownership and market competition in determining investment decisions. A seller can improve her payoff through two means: to make herself the monopolist of the product for her targeted buyer and to generate competition among alternative buyers. When the choice is between general or specific investment, the first means is achieved by choosing the specific investment and the second by choosing the general investment. The trade-off between the two means, or between the two investment types, is resolved as a function of the market structure, characterized by the numbers of sellers and buyers as well as by what assets they own.[12]

When the choice variable is the investment level, the trade-off between the two means no longer exists. Both means are fulfilled by the choice of the same specific investment. Although the benefit from the first means — to become a monopolist — is constrained by the availability of the targeted partner, a seller may enhance her payoff if she can make use of the second means to enhance her outside option. Hence, when there are fewer sellers than buyers, the second means may motivate sellers who own more assets to choose higher, more appropriate investment levels.

Our result of the harmful effect of ownership generalizes related results in Chiu, de Meza and Lockwood, and Rajan and Zingales in an important dimension. In those papers, the outside market was assumed to be fixed, independent of the conceived parties. The emphasis was usually on the investment incentives of both parties in a bilateral relationship. That is, the strategic relationship was among parties who would trade/cooperate ex post. While allowing for a multilateral relationship in which several agents work in a single firm, Rajan and Zingales still retain the feature that these agents know ex ante that they will cooperate ex post. This is not the case here. Here, all investors will never benefit from working together as a firm, and the strategic interactions occur indirectly through the market.

Our basic model can be embedded into more elaborate environments. One extension is to allow for buyers' investments, in addition to sellers' invest-

[12]The above two motives are independently pointed out in a paper by Nicita (1999), who calls the scenario in which sellers face multiple buyers and have multiple investment choices "cross competition."

ments. If buyers make investments subsequent to bargaining, then clearly the investments made will be constrained efficient and there will be little change to our basic results. Another extension is to relax the "unit trade" assumption where an investment can be used by no more than one buyer. In this case, sellers will have an extra incentive to make general investments. We think that, even if the exact condition will be different, the insights that stronger competition makes it more likely for sellers to make specific investment and that loss of ownership makes sellers more likely to invest in specific investments will still hold. The third extension would be to take into account the welfare of consumers who subsequently purchase from downstream producers (the buyers in our model). This will endogenize the joint surplus of each type of investment. While adding a dose of realism to our model, this will blur the prediction because of the externality of investments on consumers. While the roles of competition and ownership in determining the investment choices should still prevail, the normative analysis may vary drastically. Perhaps because of this, in the property rights and incomplete contracting literature, consumers' welfare is usually ignored, just as we have done in this paper. The fourth extension is to allow sellers to choose both the level of general investment and the level of specific investment. We conjecture that results similar to those found in this paper will prevail in this more general model.

Appendix A: Sketch of the Proof of Lemma 1.

Here we consider the case of E^1; the proofs for the cases of E^2 and E^3 are similar. Since efficient allocation of duplicated and nonduplicated investments can be routinely conducted, we focus on general investment here. Consider the following algorithm. (i) Assign the general investments of sellers in S_1^i to B_1^i, $i = 1, 2, 3$. (ii) In case of unfulfilled demand by any buyer in B_1^3, assign the buyer an allocated general investment in S_1^2. (iii) If still there is unfulfilled demand by any buyer in B_1^3 or B_1^2, assign the buyer an unallocated general investment in S_1^1. Then the final allocation is efficient, and more importantly we can show that the number of unallocated general investment in S_1^1 equals E^1 if E^1 is positive and equals 0 otherwise.

Case 1. $E^1 > 0$. According to above algorithm, an efficient allocation is resulted in which E^1 sellers in S_1^1 do not have their investments allocated while all buyers in $\cup_i B_1^i$ obtain general investments. Since in any other efficient allocation the same total production value is resulted, all buyers in $\cup_i B_1^i$ must obtain general investments while some seller in $\cup_j S_1^j$ must have her investment unallocated.

Case 2: $E^1 = 0$. According to the algorithm, no general investments in S_1^1 are left unallocated. It is straightforward to show that all buyers in $\cup_i B_1^i$

obtain general investments in this efficient allocation (call it ϕ_1). The same must hold in any other efficient allocation. We now show that all sellers in S_1^1 have their general investments allocated in any other efficient allocation (call it ϕ_2). Suppose, on the contrary, some seller in S_1^1 has her general investment unallocated under ϕ_2. Since the same number of buyers who obtain general investments under ϕ_2 is the same under ϕ_1, a seller in $S_1^2 \cup S_1^3$ with her general investment unallocated under ϕ_1 has it allocated under ϕ_2. For this seller to be in S_1^2, it must be that $\Delta^2 > 0$ (otherwise, according to the algorithm, her investment would be assigned to some buyer under ϕ_1 according to the algorithm). But if $\Delta^2 > 0$, for $E^1 = 0$ and $\Delta^1 \geq 0$ to hold, it must be the case that Δ^3 is negative and has an absolute value no less than that of Δ^2. Thus according to the algorithm, the putative seller in Δ^2 would have her investment allocated under ϕ_1. A contradiction. If that seller is in S_1^3, then it must be that $\Delta^3 > 0$ (otherwise, her investment would be assigned to some buyer under ϕ_1 according to the algorithm). Since $E^1 = 0$ and $\Delta^1 \geq 0$, this implies $\Delta^1 + \Delta^2 = 0$, i.e., all buyers' demand in B_1^1 and B_1^2 are satisfied—as is the case under ϕ_2—only if all sellers in S_1^1 and S_1^2 have all their investments allocated under ϕ_2. This is contradictory to the claim that under ϕ_2 some seller in S_1^1 has her investment unallocated.

Case 3: $E^1 < 0$. According to the algorithm, no general investment in S_1^1 is left unallocated, it can also be verified the number of buyers in $\cup_i B_1^i$ who do not obtain a general investment is $|E^1|$. Then there must be some buyer in $\cup_j B_1^j$ without general investment for any efficient allocation, because the number of buyers who obtain general investments under that efficient allocation must be the same as under ϕ_1. What is left to show is under any arbitrarily different efficient allocation, no general investment in S_1^1 is left unallocated. But this must be the case since there are buyers without general investments who want to match with any such sellers.

Appendix B. Proof of Lemma 2.

The first part of the proposition is easy to prove. Result (a) of the second part is trivial. We now show result (b), and proofs of the rest are similar. Suppose the result in (b) is not true, and there exists an allocation in the core such that some seller in S_1^i (say s') successfully sells its general investment at $p > 0$. Note that $E^i > 0$ implies excess supply of general investments to B_1^i. Somebody's (say, s'') general investment would be redundant. Now consider a subcoalition which includes every agent but s'. The worth of this subcoalition would be the same as the grand coalition, denoted by v. The reason is that the general investment sold by s' is now replaced by that of s'' and becomes reductant. As the initial allocation assigns a sum of payoffs to members of the subcoalition of $v - p < v$, the subcoalition can then profit by breaking away. It is thus contradictory to the claim that the original allocation is in the core.

Appendix C: A Sketch of Proof of the Bargaining Solution as a Subgame Perfect Equilibrium Outcome of an Alternating Offer Game.

Proposition A.1. *There exists a subgame perfect equilibrium outcome, which is reached immediately without delay and which approaches our bargaining solution as δ approaches unity.*

Before we describe the equilibrium strategy, some notation is in order. Let E^{it} be the E^i prevailed in substage t, and S_j^{it} (B_j^{it}) be the corresponding S_j^i (B_j^i) still in the market at the beginning of substage $t, i = 1, 2, 3, j = 1, 2, 3$. Let s_{jk}^{it} (b_{jk}^{it}) be the kth seller in S_j^{it} (kth buyer in B_j^{it}), $i = 1, 2, 3, j = 1, 2, 3$. In an order-preserving manner, identify each player unambiguously so as to keep check of the transition from substage t to substage $t + 1$. (For instance, if the kth seller in S_j^{it} survives in stage $t + 1$ but all sellers with ranks smaller than kth do not, then call this surviving seller as the first seller in $S_j^{i,t+1}$.)

We first define two sets of prices $\widetilde{p}_j^i$ and $\overline{p}_j^i, i = 1, 2, 3, j = 1, 2, 3$ as follows (superscript t is omitted):

Prices proposed by sellers in S_j^i

	$\widetilde{p}_1^i$	$\widetilde{p}_2^i$	$\widetilde{p}_3^i$
$E^i > 0$	0	$\min\{M - m, M/(1+\delta)\}$	0
$E^i = 0$	$m/(1+\delta)$	$M/(1+\delta)$	0
$E^i < 0$	m	$M/(1+\delta)$	0

Prices proposed by buyers in B_j^i

	$\overline{p}_1^i$	$\overline{p}_2^i$	$\overline{p}_3^i$
$E^i > 0$	0	$\min\{M - m, M/(1+\delta)\}$	0
$E^i = 0$	$\delta m/(1+\delta)$	$\delta M/(1+\delta)$	0
$E^i < 0$	m	$\delta M/(1+\delta)$	0

Table A1

The on-the-equilibrium-path strategy prescription is as follows. P1a (1 and 2) describes what each player does when being a proposer, and P1b (3 to 10) describes what each player does when being a responder after P1a has just been played. (To abuse the notation a little bit, we ignore the superscript t in the following description.)

1. When sellers propose, s^i_{jk} publicly proposes to sell his investment to anybody at $\widetilde{p}^i_j$, $k = 1, 2, .., |S^i_j|$; $i = 1, 2, 3$; $j = 1, 2, 3$.

2. When buyers propose, b^i_{jk} proposes $\overline{p}^i_j$ for any general investment (for $j = 1$) and for the investment specific to himself (for $j = 2, 3$), $k = 1, 2, .., |B^i_j|$; $i = 1, 2, 3$; $j = 1, 2, 3$.

3. b^i_{2k} accepts to buy from the seller who has a specific investment for him where $k = 1, 2, 3, ...,$ and $i = 1, 2, 3$. Buyer b^i_{3k} accept to buy with equal probability from the sellers who have duplicate specific investments where $k = 1, 2, 3, ...,$ and $i = 1, 2, 3$.

4. b^1_{1k} first accepts to buy from sellers in S^1_1 in order (i.e., b^1_{1k} buying from s^1_{1k} in S^1_1, etc.), $k = 1, 2, ...$ If $|S^1_1| < |B^1_1|$, excess buyers each ($k = |S^1_1| + 1, |S^1_1| + 2, ...$) accept to buy from sellers in S^1_1 independently and randomly with equal probability.

5. b^2_{1k} first accepts to buy from sellers in S^2_1 in order, $k = 1, 2, ...$ If $|S^2_1| < |B^2_1|$, excess buyers each ($k = |S^2_1| + 1, |S^1_1| + 2, ...$) accept to buy in order from sellers in S^1_1 whose offers are not accepted as prescribed by (4). The remaining excess buyers accept to buy independently and randomly with equal probability from all sellers whose offers are supposed to be accepted as prescribed above by (5).

6. In the same manner as (4) and (5), b^3_{1k} first accept to buy from sellers in S^3_1 in order, $k = 1, 2, ...$ The excess buyers accept to buy in order from sellers in S^2_1 whose offers are not accepted as prescribed in (4) and (5). The remaining buyers accept to buy in order from sellers in S^1_1 whose offers are not accepted as prescribed by (4) and (5). The further remaining buyers accept to buy independently and randomly with equal probability from sellers whose offers are supposed to be accepted as prescribed above by (6).

7. s^i_{jk} accepts to sell to the buyer for whom the seller has a specific investment where $k = 1, 2, 3, ..., j = 2, 3$, and $i = 1, 2, 3$.

8. s^3_{1k} accepts to sell to buyers in B^3_1 in order (i.e., s^3_{1k} sells to b^3_{1k} in B^3_1, etc.), $k = 1, 2, ...$ If $|S^3_1| > |B^3_1|$, excess sellers ($k = |B^3_1| + 1, |B^3_1| + 2, ...$) accept to sell to all buyers in B^3_1 independently and randomly with equal probability.

9. s^2_{1k} first accepts to sell to buyers in B^2_1 in order, $k = 1, 2, ...$ If $|S^2_1| > |B^2_1|$, excess sellers ($k = |S^2_1| + 1, |S^2_1| + 2, ...$) each accept to sell in order

to buyers in B_1^1 whose offers are not accepted as prescribed by (8). The remaining excess sellers accept to sell independently and randomly with equal probability to all buyers whose offers are supposed to be accepted as prescribed above by (9).

10. In the same manner as (8) and (9), s_{1k}^1 first accepts to sell to buyers in B_1^1 in order, $k = 1, 2, 3, \ldots$. The excess sellers accept to sell in order to B_1^2 whose offers are not accepted as prescribed by (9). The remaining sellers accept to sell in order to B_1^3 whose offers are not accepted as prescribed by (8) and (9). The further remaining sellers then accept to sell randomly and independently with equal probability to all buyers whose offers are supposed to be accepted as prescribed above by (10).

It is straightforward to verify the following two lemmas.

Lemma A.2. *Given P1a and P1b are played in stage t, then $E^{i,t} \gtreqless 0 \Rightarrow E^{i,t+1} \gtreqless 0$.*

Lemma A.3. *Given P1a and P1b are to be played in stage t and thereafter, the game will end immediately with prices as stipulated in Table A1.*

It is a bit tedious to verify that off the equilibrium path strategies can be constructed in such a way that, while they are best responses, no one can gain by unilateral deviations from P1a or P1b.

Lemma A.4. *Given that P1 is played in stages after t, there does not exist a beneficial unilateral deviation from P1a or from P1b.*

Given Lemmas A2 to A4, Proposition A.1 is immediate. In order to prove that P1a and P1b will be played hereafter, one must show that beneficial unilateral deviations are impossible, (i) when simultaneously all other players follow P1a, and (ii) when simultaneously all other players follow P1b. Case (ii) is easiest to see and is apparent from Lemma A.3. For case (i), we should consider two types of deviations: less aggressive and more aggressive. A less aggressive deviation is one in which upon accepting the offer the payoff to the deviator is less than what P1a and P1b allow him (hence, a lower (greater) asking price when the deviator is a seller (buyer)). A more-aggressive deviation is just the opposite of a less aggressive deviation. It refers to an asking price greater (lower) than the deviator's asking price as prescribed by P1a when the deviator is a seller (buyer). That a less aggressive deviation can never be beneficial can be shown easily. Basically, the deviation can be beneficial only when the offer is *not accepted*, and in the next stage the competition becomes more favorable to the deviator so that he or she can obtain a greater payoff even after discounting. This can be shown to be impossible, for it will happen only if somebody else makes a strictly dominated decision in stage t.

To show that a more aggressive deviation can never be beneficial, one needs to stipulate the response strategy prescription—which we call P2—to any of such unilateral deviations from P1a. P2 is basically the same as P1b, except for the responder who is supposed to accept the deviator's candidate equilibrium proposal prescribed by P1b. Depending on the case, that responder is prescribed either to reject any offer or to accept some other offer. By so doing, this ensures that, if the responder survives the next stage, he will see a competition environment similar to the one in the last stage ($E^{i,t+1} = E^{i,t}$). In this case his payoff from stage $t+1$ (given that P1a and P1b are to be played) will be definitely greater than that from accepting the deviating offer in stage t. The last thing to show is that no unilateral deviations from P2 will be beneficial given that P1a and P1b are to be played hereafter. This again is indeed true. All of the above altogether shows Proposition A.1.

Appendix D. Proof of Proposition 2

This is equivalent to showing that in every equilibrium $E^i = 0$ for all i. Suppose otherwise $E^k \neq 0$ for some k; according to Lemma 2, we have $E^k < 0$. Using the definition of E^k, it must be the case that $\Delta^l < 0$ for some l. Since $|S^l| \geq |B^l|$, $|S^l| = \sum_{j=0}^{4} |S_j^l|$, and $|B^l| = \sum_{j=1}^{3} |B_j^l|$, we have

$$\sum_{j=0}^{4} |S_j^l| \geq \sum_{j=1}^{3} |B_j^l|,$$

implying

$$|S_0^l| + |S_1^l| + |S_2^l| \geq |B_1^l| + |B_2^l|$$

after taking into account of part 1 of Lemma 2. With some manipulation, we have

$$\Delta^l \geq |B_2^l| - |S_0^l| - |S_2^l|.$$

Therefore the fact that $\Delta^l < 0$ implies that either $|S_0^l|$ or $|S_2^l|$ must be positive. We now argue that, given that E^k is negative, any member in S_0^l could have improved her payoff by making a general investment, and likewise any member in S_2^l could have improved her payoff by switching to a general investment. In the former case, both Δ^l and E^k are increased by one as a result of the unilateral deviation. If the new E^k is still negative, the deviating seller will receive a price of m; if it is now zero, she will receive a price of $m/2$. Either case, the deviation is strictly beneficial. In the latter case, since there is no change to Δ^l and E^l as a result of the deviation, the deviating seller will certainly obtain a price of $m > M/2$. But there should not be unilateral beneficial deviation in an equilibrium. A contradiction. Hence it must be true that $E^i = 0$ for all i.

Appendix E. Proof of Proposition 3. (where $|S| < |B|$)

We first note that in any equilibrium of the game, it must be true that $E^3 = 0$, whose proof is similar to the one in Proposition 2 and is omitted here. Note that $E^3 = 0$ implies $\Delta^3 = 0$. Together they imply that $S^3 = S_2^3$, i.e., every seller in S^3 chooses a nonduplicate investment for some buyer in B^3. We next argue that $E^1 < 0$ and $E^2 < 0$. Recall a few relationships:

$$\begin{aligned}
|S_2^1| + |S_2^2| + |S_2^3| &= |B_2^1| + |B_2^2| + |B_2^3| \\
|S_0^1| + |S_1^1| + |S_2^1| &= |B_1^1| + |B_2^1| \\
|S_0^2| + |S_1^2| + |S_2^2| &< |B_1^2| + |B_2^2|
\end{aligned}$$

where the first equation is a restatement of (1), the second and third equations are statements of $|S^1| = |B^1|$ and $|S^2| < |B^2|$, respectively (after taking into part 1 of Lemma 2). Using the last two equations, we have

$$\begin{aligned}
\Delta^1 + \Delta^2 &< \left(|B_2^1| + |B_2^2|\right) - \left(|S_2^1| + |S_2^2|\right) - \left(|S_0^1| + |S_0^2|\right) \\
&= |S_2^3| - |B_2^3| - \left(|S_0^1| + |S_0^2|\right) \quad (\because \text{the first equation})
\end{aligned}$$

Since $|S_2^3| = |B_2^3|$ (every seller in S^3 makes a nonduplicate investment for some buyer in B^3), we have $\Delta^1 + \Delta^2$ must be negative, implying also that either Δ^1 or Δ^2 or both are negative. Coupled with $\Delta^3 = 0$, this implies that $E^1 = \Delta^1 + \min\{0, \Delta^2\} < 0$. The negativity of E^1 implies that all sellers in S^1 would choose a general investment, leading to $\Delta^1 = 0$. As a consequence Δ^2 is negative and so is E^2. Given $E^2 < 0$, all sellers in S^2 will choose general investments. And this completes the proof.

Appendix F. Proof of Proposition 4

(1) We show this by contradiction. Suppose two sellers in S^i, s_1 and s_2, make a general and specific investment, respectively. Given the investment choices of all other sellers, we can determine that one of the following must be true: (i) $E^i > 0$, (ii) $E^i = 0$, and (iii) $E^i < 0$. If (i) is the case, then s_1, who made a general investment will get a zero payoff in ex post bargaining. She of course could have done better. A contradiction. If (ii) is the case, then s_1 and s_2 sell at a price of $m/2$ and $M/2$, respectively. Foreseeing this, s^1 should

have switched to make a specific investment for the one who is supposed to purchase the general investment from her in the punitive equilibrium. In this case, it can be checked that E^i will remain the same, and s_1 will then receive $M/2$, rather than $m/2$. A contradiction. If (iii) is the case, s_1 and s_2 receive m and $M/2$ respectively. Foreseeing this, s_2 should have switched to make a general investment to receive a payment of m, rather than $M/2$ (with such a switch, E^i will remain to be negative). Hence, a contradiction.

(2) Suppose some s^i in S^i makes a specific investment but some s^{i+1} in S^{i+1} makes a general investment. Clearly, with such a specific investment, s^i must be facing a situation where $E^i \geq 0$. Otherwise (i.e., $E^i < 0$), s^i should have switched to a general investment to obtain a payment of m, which is greater than $M/2$, her payoff from a specific investment. Note that such a switch of investment does not alter the value of $E^i > 0$. It is thus a contradiction. There are now two subcases to consider. Either (i) $E^i > 0$ or (ii) $E^{\ i} = 0$. In the former case, from Remark 3, $E^{i+1} \geq E^i > 0$, s^{i+1}'s equilibrium selling price of her general investment would be zero, and she cannot even get back her investment cost. This clearly is contradictory. In the latter case, if $E^{i+1} > E^i = 0$, then the aforementioned contradiction still occurs. If $E^{i+1} = 0$, then seller s^{i+1}'s gross gain is $m/2$ from a general investment. But foreseeing this, she should have switched to a specific investment, whereby E^{i+1} will remain the same, and she can obtain a greater gross gain of $M/2$. A contradiction.

(3)We first show that $e^i \geq 0$ implies that every investing seller in S^i chooses a nonduplicate specific investment. Consider the case where $i = 3$. Suppose, by contrary, that in equilibrium some seller in S^3 chooses a general investment. Equilibrium condition entails that all investing sellers (in all S^i) must make general investment. The fact that investing sellers in S^3 choose a general investment also implies that $E^3 < 0$, in turn implying $\Delta^3 < 0$. It is easy to see that no seller in S^i will be noninvesting (otherwise, such a noninvesting seller would have benefitted from a unilateral deviation to a general investment for sale to some buyer in B_1^3 who does not get any investment). As a result, $\Delta^i \equiv |S_1^i| - |B_1^i| = |S^i| - |B^i| \equiv d^i$ for all i and $E^3 = e^3 \geq 0$, contradictory to the earlier claim that $E^3 < 0$.

The same argument works for the case where $i = 2$. Suppose $e^2 \geq 0$ but, by contrary, in equilibrium some seller in S^2 chooses a general investment. Then equilibrium conditions entail that all investing sellers in S^1 and S^2 make general investment. The fact that investing sellers in S^2 choose a general investment also implies that $E^2 < 0$ so that a general investment is sold at m. Then every seller in S^1 and S^2 invests (otherwise, such a noninvesting seller would have benefitted from a unilateral deviation to a general investment for sale to some buyer in $B_1^2 \cup B_1^3$ who does not get any investment). As a result,

$\Delta^i \equiv |S_1^i| - |B_1^i| = |S^i| - |B^i| \equiv d^i$ for $i = 1, 2$. Therefore e^2 and E^2 differ in their last terms, and we argue that through clarifying the relationship of these last terms we establish that $E^2 = e^2$ which is a contradiction. To see this, suppose some general investment by S^1 and S^2 is sold to some buyer in B^3 in equilibrium. Since the price is m, it must be the case that $E^3 < 0$ and every investment seller in S^3 must choose general investment and every seller in S^3 invests (in case of a noninvesting seller, she can choose specific investment and benefit from such a deviation). Hence, $\Delta^3 = |S_1^3| - |B_1^3| = |S^3| - |B^3| \equiv d^3$ and $E^2 = e^2$. On the other hand, suppose no general investment by S^1 and S^2 is sold to any buyer in B^3 in equilibrium. This occurs only when $\max\{0, d^1\} + d^2 = 0$ and $d^3 \geq 0$ (using the definition of e^2 and the fact that it is not negative). Since no investment from $S^1 \cup S^2$ will be sold to buyers in B^3, it follows that $\Delta^3 = 0 = \min\{0, d^3\}$ and $E^2 \equiv e^2$. Hence, either case, there is a contradiction.

Suppose $e^1 \geq 0$ but, by contrary, in equilibrium some seller in S^1 chooses a general investment. Then equilibrium conditions entail that all investing sellers in S^1 make general investment. The fact that investing sellers in S^1 choose a general investment also implies that $E^1 < 0$ so that a general investment is sold at m. Then every seller in S^1 invests (otherwise, such a noninvesting seller would have benefitted from a unilateral deviation to a general investment for sale to some buyer who does not get any investment). As a result, $\Delta^1 \equiv |S_1^1| - |B_1^1| = |S^1| - |B^1| \equiv d^1$. Suppose some general investment by S^1 is sold to some buyer in $B^2 \cup B^3$ in equilibrium. Since the price is m, it must be the case that $E^i < 0$ for some $i = 2, 3$ and every investment seller in S^i must choose general investment. This contradicts the fact that since $e^i \geq e^1 \geq 0$ sellers in S^i should make specific investments. Suppose, on the other hand, no general investment by S^i is sold to any buyer in $B^2 \cup B^3$ in equilibrium. This occurs only when $\Delta^1 = 0$ and either Δ^2 or Δ^2 or both are negative (using the definition of $E^1 < 0$ and that $\Delta^1 = d^1$ is nonnegative given $e^1 \geq 0$). As a consequence $E^2 = E^1 < 0$, in which case sellers in S^2 should choose general investments, contradicting that under $e^2 \geq 0$ (which is the case) sellers in S^2 should make specific investments. In other words, either case, there is a contradiction.

(4) We now show that $e^i < 0$ implies that every investing seller in S^i chooses a general investment. We start with the case where $i = 1$. Suppose $e^1 < 0$ but, by contrary, in equilibrium some seller in S^1 chooses a specific investment. To be an equilibrium, it must be the case that every investing seller in S^1, S^2 or S^3 choose specific investment and $E^i = 0$ for $i = 1, 2, 3$, implying $\Delta^i = 0$ and $|B_1^i| = |S_1^i| - \Delta^i = 0$ for $i = 1, 2, 3$. In other words, every buyer in B^i has a nonduplicate specific investment made for him, implying

$d^1 \geq 0, d^1 + d^2 \geq 0$, and $d^1 + d^2 + d^3 \geq 0$. Since d^1 is positive, $e^1 < 0$ requires that $d^2 + \min\{0, d^3\} < 0$ and hence $e^1 = d^1 + d^2 + \min\{0, d^3\}$. If $d^3 \geq 0$, then $e^1 = d^1 + d^2 \geq 0$; if $d^2 < 0$, then $e^1 = d^1 + d^2 + d^3 \geq 0$. Either case, it is a contradiction to the hypothesis $e^1 < 0$.

We next show the case for $i = 2$. Suppose $e^2 < 0$. We first note that all investing sellers in S^1 chooses general investment since $e^1 \leq e^2 < 0$ and we have proved that the investment decisions for this case in the preceding paragraph. In addition, $E^1 < 0$ and there are no non-investing sellers in S^1. Recall our hypothesis that $e^2 < 0$, which implies $d^2 + \min\{0, d^3\} < 0$. As a result $\Delta^1 = d^1$. If $d^1 > 0$, some general investment in S^1 is sold to some buyer in B^2 or B^3, leading to either $E^2 < 0$ or $E^3 < 0$, both implying that investing sellers in S^2 make only general investments. If $d^1 \leq 0$, some seller in S^2 chooses a specific investment. To be an equilibrium, it must imply that all investing sellers in S^2 and S^3 make specific investment and $E^2 = E^3 = 0$, implying $\Delta^2 = \Delta^3 = 0$. Since no sellers in S^2 and S^3 have made general investments, $|B_1^i| = |S_1^i| - \Delta^i = 0, i = 2, 3$. That is, every buyer in S^2 and S^3 has a nonduplicate specific investment made for him, i.e., $d^2 \geq 0$ and $d^2 + d^3 \geq 0$, implying that $d^2 + \min\{0, d^3\} \geq 0$. But this is in contradiction to the hypothesis that $e^2 < 0$ which implies $d^2 + \min\{0, d^3\} < 0$.

Finally, we come to the case where $i = 3$. Suppose $e^3 < 0$. Since $e^1 \leq e^2 \leq e^3$, we have shown that all investing sellers in S^1 and S^2 make general investments. In addition $E^1 = E^2 < 0$, each general investment is sold at a price of m, and there are no non-investing sellers in S^1 and S^2. This implies that $\Delta^i \equiv |S_1^i| - |B_1^i| = |S^i| - |B^i| \equiv d^i$, for $i = 1, 2$. Consider Δ^3. Since no specific investments will be supplied from $S^1 \cup S^2$, we have $|S_2^3| = |B_2^3|$ and therefore $\Delta^3 \equiv |S_1^3| - |B_1^3| = (|S_1^3| + |S_2^3|) - (|B_1^3| + |B_2^3|) \leq |S^3| - |B^3| = d^3$. Therefore, $E^3 \leq e^3 < 0$, and investing sellers in S^3 should indeed choose general investments. This completes the whole proof.

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