Uncertainty, Arbitrage and Intra-Industry Trade*

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Abstract

When firms in the same industry located in different regions or countries experience shocks to production costs in their respective industries that are imperfectly correlated, arbitrage opportunities automatically lead to trade. Trade can either stabilize or destabilize the price faced by producers in a given country. Producers' surplus is affected due to the "variance-covariance" effect, while consumers' surplus is more directly affected through the variance of the product price. The paper examines how consumers' surplus, producers' surplus and social welfare are affected when the regions switch from autarky to free trade in the presence of industry and region-specific cost shocks. Contrary to Anderson et al. (1989) and Moner-Colonques (1998), under Cournot competition, when the industries are symmetric in the two regions, producers' surplus can increase in both regions in the switch from autarky to trade. In general, depending on the variance of the cost shocks in the two regions, the correlation coefficient between the shocks, and the number of firms, producers' and consumers' surplus in a given country can be either higher or lower under trade compared to autarky. However, social welfare is higher in both regions under a surprisingly robust set of conditions. Contrary to traditional trade theory, the gain in social welfare in several situations is due to the gains in producers' surplus offsetting the loss in consumers' surplus.

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1. Introduction

Traditional trade theory implies that one of the major benefits associated with the movement of goods between regions is due to greater specialization in accordance with the principle of comparative advantage. Recent trade theory, on the other hand, notes that a large volume of trade takes place between regions with similar resource endowments and is intra-industry in nature. Intra-industry trade is explained in terms of increasing returns to scale, and one of the main benefits of intra-industry trade is greater product variety enjoyed by consumers.

In this paper, we analyze the welfare consequences of intra-industry trade between regions when the regions are subject to imperfectly correlated production or cost shocks. Examples of such shocks could be weather uncertainties affecting agriculture and agro-based industries, interruptions in supply such as the recent power outages in California, labor disputes, changes in commodity tax rates, oil price shocks, and, for countries, exchange rate changes that affect the cost of imported inputs, or macro-economic shocks that affect wages and prices in the economy.

Trade between regions would occur in the presence of these shocks for a simple reason: imperfectly correlated regional shocks would result in price differentials in local markets which present arbitrage opportunities. Goods would move from regions with low prices to those with high prices and in the process reduce the price divergence between regions. Somewhat surprisingly, the welfare consequences of such arbitrage have not received much attention in the trade literature. One notable exception is Newbery and Stiglitz (1981), who argue that trade motivated by such arbitrage opportunities can be Pareto inferior if both producers (farmers in their model) and consumers are risk-averse.

For Newbery and Stiglitz (1981), the focus was the effect of stabilization policies on the welfare of farmers and the assumption of risk-aversion is very reasonable in that context. However, when manufacturing firms in the same industry in different regions are subject to shocks which affect all firms in the industry in the same region but not in the other region (i.e. the shocks are industry and region-specific), then risk-neutrality may be a better assumption. The shareholders of the firms should be able to diversify away the shocks that are specific to their industries and therefore can be treated as effectively risk-neutral. One then has to address the welfare issues from this standpoint.[1]

[1] Moreover, the source of uncertainty in Newbery and Stiglitz (1981) is random firm output. In our model, output will be chosen after (cost) uncertainty is realized. As will be clear below, this has important implications for the impact of uncertainty and trade on the expected producers' surplus.

Existing empirical evidence shows that price differentials dissipate reasonably quickly within regions in the same country, but not so across borders (see, for example, Parsley and Wei (1996) for evidence on the speed of convergence of prices in U.S. cities, and Engel and Rogers (1996) for the so-called "border effect" between prices in U.S. and Canadian cities across the U.S.-Canada border). The reasons for the border-effect are not very well understood as yet; however, we believe that it is important to ask the normative question as to whether market integration – if it could be achieved – is desirable, and what its possible impact might be on consumers and producers of the regions. It is also important to note that price convergence has been a key issue surrounding the European Economic and Monetary Union (EMU). The President of the European Central Bank stated recently:

> Price level convergence could be expected to take place in the euro area for at least two reasons. First, the completion of the internal market and increased cross-border price transparency contribute to eroding the scope for the existence of substantial price differentials for products which are easily tradable across borders. To a large extent, this may have taken place already before the start of Economic and Monetary Union (EMU), but differences remain. One example of such a price convergence that has attracted public attention relates to car prices. Secondly, with regard to goods and services which are less easily tradable across national borders (such as housing and hairdressing), the long-term convergence of productivity and living standards across the euro area would create a tendency towards price level convergence.[2]

The issue of how, in the presence of oligopolistic competition, the opening up of trade affects intra-industry trade flows and the welfare of producers, consumers and society, has been addressed in a number of papers (see, for example, Markusen (1981), Brander and Krugman (1983), Helpman (1984), Helpman and Krugman (1985)). Helpman and Krugman (1985) show that when countries differ in more than one respect, the direction of trade flow is not necessarily from the country with low pre-trade price to the one with higher pre-trade price. They also show that opening up of trade has a "pro-competitive" effect, leading to a reduction in monopoly distortions and gains from trade over and above the competitive model. Anderson, Donsimoni and Gabszewicz (1989) address the impact of opening up of trade (or, equivalently in their models, market integration) on the

[2] Speech by Willem F. Duisenberg, President of the European Central Bank, at the Financial Services Industry Association, Dublin, 6 September, 2000.

profits of firms in oligopolistic industries. Anderson et al.(1989) point out that there are two offsetting effects of market integration. Firms in each region gain from selling their products in foreign markets. However, they also face greater competition from the output of firms in the other region. Anderson et al. (1989) find that in general, the total profits of firms from at least one region will decrease, and if the regional markets are symmetric, firms in both regions will attain lower profits.

The main difference between the partial equilibrium framework in some of the papers mentioned above and ours is in the introduction of uncertainty. In Helpman and Krugman's (1985) framework, for example, there will be no trade if the countries are otherwise symmetric (however, the possibility of trade will still have a "pro-competitive effect"). In our framework, even if the countries are otherwise identical, the realization of the random cost shock may be different, and thus trade will occur. The ex-ante welfare consequences of trade when markets are opened up to eliminate price differentials is the main focus of our paper. We find that in the presence of uncertainty, there are additional welfare effects over and above the "pro-competitive" effect that the existing literature has mainly emphasized. If the variance of the cost shocks is sufficiently large, the additional effects will dominate the "pro-competitive" effects.

Moner-Colonques (1998), like us, also introduces cost uncertainty. Each firm has a cost shock that is private information to that firm, and it knows the distribution from which other firms' cost shocks are drawn. Moner-Colonques (1998) finds that if the variance of the cost shock is sufficiently high, firms from one of the regions will benefit. However, it is never possible, in the symmetric case, for firms in both regions to benefit. Our framework has two main differences with Moner-Colonques (1998).[3] First, we assume that firms within the same region experience the same shock, but these shocks are imperfectly correlated across regions. Second, we do not assume that the shocks are private information – in fact, the shocks are assumed to be common knowledge and the output decisions are taken after the shocks are realized. We find that in the symmetric case, as long as the cost shocks across regions are not perfectly correlated, a sufficiently high variance of the shock would increase the profits of firms in *both* regions under trade (or market integration)[4]

[3] As in Moner-Colonques (1998), our analysis is also carried out in a framework of linear demand and marginal cost curves.

[4] It is useful to point out here that for the particular demand functions considered in this paper and in Moner-Colonques (1998), "trade" (referring , in the above two papers, to a situation where firms in each region are free to sell in either market, but the markets are segmented in the sense that consumer arbitrage is not possible) and "market integration" (referring to a situation where a single price prevails in an integrated market) imply the same equilibrium. See Anderson et. al. (1989), page 731.

compared to autarky. This is different from the results of both Anderson et al. (1989) as well as Moner-Colonques (1998). Moreover, unlike these two papers, we also carry out a full welfare analysis. We find that in the symmetric case, social welfare improves from market integration in both regions. Even in the asymmetric case, the conditions required for social welfare to decrease in any country are rather stringent. Further, social welfare must be higher in at least one region.

To understand the way in which trade can change the exposure to uncertainty faced by producers and consumers and thus affect their welfare, it is useful to consider a situation in which the "pro-competitive" effect is absent, i.e. firms are price takers. Accordingly, our analysis begins with the case of price-taking firms. In the absence of trade, the country-specific cost shocks induce variability in the industry price. We show that the impact of variability of price on expected profit can be decomposed into two terms: the variance of price, which affects the expected profit of firms positively, and the covariance of the price with the cost shock faced by the firm, which affects its expected profit negatively (the collective effect is referred to as the "variance-covariance effect" in the subsequent analysis). Trade affects the variability of the price as the price is now a function of the cost shocks of both regions. However, trade also reduces the covariance between the price and a firm's own cost shock. Trade affects the welfare of consumers more directly because the indirect utility function of consumers is quasi-convex in prices, so that consumers prefer a more variable price to a less variable one. We find that social welfare is higher with trade than under autarky for each region, irrespective of the variance of the shocks.

One interesting difference of our findings with the existing literature is that although the classical gains from trade result holds for both countries in general, the internal distribution of gains and losses is reversed. Both traditional trade theory based on perfect competition and new trade theory based on imperfect competition show that the gains from trade are driven by gains to consumers. In our setting, however, the gain in social welfare in several situations is due to the gains in producers' surplus offsetting the loss in consumers' surplus.

The rest of the paper is organized as follows. The basic model is introduced in section 2. We then analyze how trade affects the welfare of consumers and producers and the social welfare under perfect competition (section 3) and Cournot competition (section 4). Finally, we conclude in section 5. Some of the detailed derivations are relegated to the appendix.

2. The Basic Model

We consider two countries[5], Home and Foreign, which are characterized by an identical number of firms, n, producing a homogeneous product. Each domestic firm has the following cost function:

$$C(q) = \alpha q + wq + \frac{\beta}{2}q^2.$$

Similarly, each foreign firm has the following cost function:

$$C^*(q^*) = \alpha q^* + w^* q^* + \frac{\beta}{2}q^{*2}.$$

w and w^* are two random variables representing cost shocks with $E(w) = E(w^*) = 0$, $E(w^2) = \sigma^2$, $E(w^{*2}) = \sigma^{*2}$, and $Corr(w, w^*) = \rho$. Production takes place after w and w^* are realized. The demand for the good in both the Home country and the Foreign country is given by

$$Q = \frac{1}{b}(a - p).$$

The basic reason why trade affects welfare in this environment is that it affects the exposure of producers and consumers to uncertainty. Let us go one step back and try to understand first how the producers and consumers react to the exposure to uncertainty.

Since the shocks affect the marginal costs of firms, equilibrium price also depends on the realization of the cost shocks, $p = p(w)$.[6] Given w, the consumers' surplus is $CS = \frac{1}{2b}(a - p(w))^2$, and its expected value is

$$E(CS) = \frac{1}{2b}\left[a^2 + Var(p) - E(p)\left(2a - E(p)\right)\right]. \tag{1}$$

Thus, expected consumers' surplus increases with the variance of price, that is, consumers prefer the price to be more variable, and decreases with the expected price.

The effect of cost uncertainty on an individual producer's expected profit is not that straightforward; there is a trade-off between the direct effect of the cost uncertainty working through the variance of the cost shock and an indirect effect influencing the variability of price and the covariance of price with the cost shock. Given w, the expressions for profits of an individual producer under price-taking behavior and under Cournot competition

[5] For the purposes of this paper, we can use "countries" and "regions" interchangeably.

[6] Without loss of generality, we develop the argument from the point of view of the Home country.

are given by $\pi|_{\text{Price-taking}} = \dfrac{1}{2\beta}(p(w) - \alpha - w)^2$, and $\pi|_{\text{Cournot}} = \dfrac{2b+\beta}{2(b+\beta)^2}(p(w) - \alpha - w)^2$, respectively.[7] So the expressions for expected profits are

$$E(\pi|_{\text{Price-taking}}) = \frac{1}{2\beta}\left[Var(p) - 2Cov(p,w) + E(p)\left(E(p) - 2\alpha\right) + \sigma^2 + \alpha^2\right], \qquad (2)$$

$$E(\pi|_{\text{Cournot}}) = \frac{2b+\beta}{2(b+\beta)^2}\left[Var(p) - 2Cov(p,w) + E(p)\left(E(p) - 2\alpha\right) + \sigma^2 + \alpha^2\right]. \qquad (3)$$

A higher variance of the cost shock has a direct and positive effect on expected profit.[8] However, because the cost shock is a common shock to all firms in the industry, the industry price follows the cost shock. Thus, cost uncertainty exerts an indirect effect working through the product price. Clearly, the expected profit of an individual producer increases with the variance of price but decreases with the covariance between the price and the cost shock. For future reference we call this indirect effect of the cost shock on producer's expected profit (represented by the sum of the first two terms in the right hand side of equations (2) and (3)) the "variance-covariance effect". Note also that, unsurprisingly, the expected profit is also increasing in the expected price.

A move from autarky to trade affects the welfare of consumers and producers and hence the social welfare as the variability and comovements of price (with the cost shock) are now affected since the price now depends on the cost shocks of both countries. We analyze these effects under alternative assumptions about the market structure in the following two sections.

3. Price-Taking Firms

We first consider the case in which the firms are price takers. Consider autarky first. Given w, the equilibrium in the Home country requires marginal cost to equal price, and

[7] The expression for $\pi|_{\text{Price-taking}}$ is derived in section 3. To derive the expression for $\pi|_{\text{Cournot}}$, note that the ith firm's problem is: $\underset{\{q_i\}}{\text{Maximize}}\ [a - b(\sum_j q_j) - (\alpha + w + \frac{\beta}{2}q_i)]q_i$. The first-order condition for this problem is: $a - b(\sum_j q_j) - (\alpha + w + \frac{\beta}{2}q_i) = q_i(b + \frac{\beta}{2})$. Using this we can write $q_i = \dfrac{p - \alpha - w}{b + \beta}$, since $p = a - b(\sum_j q_j)$. Finally, we use the first-order condition again to get the expression for profit: $\pi_i = (b + \frac{\beta}{2})q_i^2 = \dfrac{2b+\beta}{2(b+\beta)^2}(p - \alpha - w)^2$.

[8] The intuition is that the firm facing the cost shock is able to adjust output *ex post*. If output were to be preset at the profit-maximizing level when the cost shock is at its mean value, the expected profit is the same as under certainty. However, by optimally adjusting output *ex post* in response to the realized cost shocks, the expected profit must increase.

the equality of demand and supply, that is,

$$p = \alpha + w + \beta q,$$

$$nq = \frac{1}{b}(a - p).$$

Solving for p yields

$$p|_{\text{Autarky}} = \frac{1}{\beta + bn}[a\beta + bn(\alpha + w)].$$

Given w, the equilibrium profit of a firm is

$$\pi|_{\text{Autarky}} = \frac{1}{2\beta}(p - \alpha - w)^2 = \frac{\beta}{2(\beta + bn)^2}(a - \alpha - w)^2.$$

Thus, the expected profit is

$$E(\pi|_{\text{Autarky}}) = \frac{\beta}{2(\beta + bn)^2}[(a - \alpha)^2 + \sigma^2].$$

Also, given w, the consumers' surplus is

$$CS|_{\text{Autarky}} = \frac{1}{2}(a - p)nq = \frac{bn^2}{2(\beta + bn)^2}(a - \alpha - w)^2,$$

and therefore the expected consumers' surplus is

$$E(CS|_{\text{Autarky}}) = \frac{bn^2}{2(\beta + bn)^2}[(a - \alpha)^2 + \sigma^2].$$

Thus, the expected social welfare is

$$\begin{aligned} E(SW|_{\text{Autarky}}) &= nE(\pi|_{\text{Autarky}}) + E(CS|_{\text{Autarky}}) \\ &= \frac{n}{2(\beta + bn)}[(a - \alpha)^2 + \sigma^2]. \end{aligned}$$

Now, consider trade. Given w and w^*, the equilibrium requires

$$p = \alpha + w + \beta q,$$

$$p = \alpha + w^* + \beta q^*,$$

$$n(q+q^*) = \frac{2}{b}(a-p).$$

Solving for p yields

$$p|_{\text{Trade}} = \frac{1}{\beta + bn}\left[a\beta + \frac{bn}{2}(2\alpha + w + w^*)\right].$$

Given w and w^*, the equilibrium profit of a domestic firm is

$$\pi|_{\text{Trade}} = \frac{1}{2\beta}(p-\alpha-w)^2.$$

Substituting for p, we get the expected profit from trade to be

$$\begin{aligned} E(\pi|_{\text{Trade}}) &= E(\pi|_{\text{Autarky}}) \\ &+ \frac{bn}{8\beta(\beta+bn)^2}[(4\beta+bn)\sigma^2 + bn\sigma^{*2} - (4\beta+2bn)\rho\sigma\sigma^*]. \end{aligned} \tag{4}$$

It is useful to first consider the benchmark case of $\rho = 1$. If the countries are symmetric (i.e. $\sigma = \sigma^*$), it is clear that trade will not affect either the variance of price or the covariance between price and the cost shock for a given country. Thus, no change in the expected profits from trade will result. However, matters are different if the countries are not symmetric, or if the correlation is imperfect. The following proposition (which follows from equation (4) and is proved in Appendix A.1) summarizes:

Proposition 1.

1. *The gain in expected producers' surplus under trade is decreasing in ρ for both countries.*
2. *For any value of ρ, the expected producers' surplus is higher under trade than under autarky for the country with the higher variance of the cost shock.*
3. *For $\rho < 0$, the expected producers' surplus is higher under trade than under autarky for both countries.*
4. For $\rho > 0$, if $\rho^2 < \dfrac{4bn(4\beta+bn)}{(4\beta+2bn)^2} = 1 - \dfrac{16\beta^2}{16\beta^2 + 16\beta bn + 4b^2n^2}$, *the expected producers' surplus is higher under trade than under autarky for both countries.*

5. For $\rho > 0$,if $\rho^2 \geq \frac{4bn(4\beta + bn)}{(4\beta + 2bn)^2} = 1 - \frac{16\beta^2}{16\beta^2 + 16\beta bn + 4b^2n^2}$, *the expected producers' surplus is higher under trade than under autarky for both countries if either* σ *is sufficiently close to* σ^* $(x_2 \leq \frac{\sigma}{\sigma^*} \leq x_3)$ *or* σ *is sufficiently different from* σ^* *(either* $\frac{\sigma}{\sigma^*} \leq x_1$*, or* $\frac{\sigma}{\sigma^*} \geq x_4$*).*[9]

6. *If* $\sigma = \sigma^* = \bar{\sigma}$ *and* $\rho < 1$*, the gain in expected producers' surplus under trade is increasing in* $\bar{\sigma}$*.*

Before discussing these results in more detail, some comments are in order. First, it is worth pointing out that the upper bound on ρ in Proposition 1.3 is increasing in the number of firms in each country, n, so that the higher the number of firms, the more likely it is that firms in both countries will benefit from the opening of trade. Secondly, the upper bound rapidly approaches 1 as n increases (for example, for $n = 5$, the value of the upper bound (with $\beta = b = 1$) is already 0.918), so that the condition is actually not very restrictive for price-taking industries with a large number of firms.

In order to understand the results in Proposition 1 note first that the expected price in equilibrium does not change due to the move from autarky to trade:

$$E(p|_{\text{Autarky}}) = \frac{a\beta + bn\alpha}{\beta + bn} = E(p|_{\text{Trade}}).$$

Now, from equation (2), note that, ceteris paribus, a higher variance of the cost shock (σ^2) increases expected profit. However, if the cost shock is a common shock, the product price follows the cost shock, and lowers expected profits by offsetting the impact of the variability of the cost shock. This is the "variance-covariance effect", represented by the first two terms in the bracketed expression in equation (2). Under autarky, their sum is negative.[10] Now suppose that trade opens up,[11] and consider the extreme case of $\rho = -1$. In this case, there is no fluctuation in the price. The "variance-covariance effect" is zero,

[9] x_1, x_2, x_3 and x_4 are defined in Appendix A.1. Note that $\frac{bn}{4\beta + bn} \leq x_1 < x_2 \leq 1 \leq x_3 < x_4 \leq \frac{4\beta + bn}{bn}$, and $x_1x_4 = 1$, and $x_2x_3 = 1$.

[10] We have $\left[Var(p|_{\text{Autarky}}) - 2Cov(p|_{\text{Autarky}}\,,\ w)\right] = (\frac{bn}{\beta + bn})^2\sigma^2 - \frac{2bn}{\beta + bn}\sigma^2$.

[11] With trade, we have
$[Var(p|_{\text{Trade}}) - 2Cov(p|_{\text{Trade}}\,,\ w)] = \frac{1}{4}(\frac{bn}{\beta + bn})^2(\sigma^2 + \sigma^{*2} + 2\rho\sigma\sigma^*) - \frac{bn}{\beta + bn}(\sigma^2 + \rho\sigma\sigma^*)$.

and firms in both countries are better off. When both countries have equal variance of the cost shock, the variance-covariance effect remains unchanged for $\rho = 1$, and is decreasing in ρ. Thus, trade always benefits producers relative to autarky in this case of equal variances.

For the case of unequal variances, trade may increase or decrease price variability and the covariance between the price and the cost shock for the producers, depending on whether the country has a lower or higher variance under autarky. Note that

$$Var(p|_{\text{Trade}}) - Var(p|_{\text{Autarky}}) = \frac{1}{4}\left(\frac{bn}{\beta + bn}\right)^2 \left(\sigma^{*2} - 3\sigma^2 + 2\rho\sigma\sigma^*\right), \text{ and}$$

$$Cov(p|_{\text{Trade}},\ w) - Cov(p|_{\text{Autarky}},\ w) = \frac{1}{2}\left(\frac{bn}{\beta + bn}\right)\left(\rho\sigma\sigma^* - \sigma^2\right).$$

Again, to begin with, consider the case of $\sigma = \sigma^* = \bar{\sigma}$. We get

$$\left[Var(p|_{\text{Trade}}) - Var(p|_{\text{Autarky}})\right]\Big|_{\sigma=\sigma^*=\bar{\sigma}} = -\frac{1}{2}\left(\frac{bn}{\beta + bn}\right)^2 \bar{\sigma}^2(1-\rho), \text{ and}$$

$$2\left[Cov(p|_{\text{Trade}},\ w) - Cov(p|_{\text{Autarky}},\ w)\right]\Big|_{\sigma=\sigma^*=\bar{\sigma}} = -\left(\frac{bn}{\beta + bn}\right)\bar{\sigma}^2(1-\rho).$$

Clearly, trade decreases price variability resulting in a lower producer's profit. On the other hand, trade reduces the covariance of price with the cost shock which has a positive effect on the producer's profit.

From equation (2), to determine the impact of the switch from autarky to trade on expected producers' surplus, it is enough to look at the change in $Var(p) - 2Cov(p)$ (since the expected price, $E(p)$, remains unchanged). It is immediate from the above two expressions that for $\sigma = \sigma^* = \bar{\sigma}$,

$$[Var(p|_{\text{Trade}}) - 2Cov(p|_{\text{Trade}},\ w)] > \left[Var(p|_{\text{Autarky}}) - 2Cov(p|_{\text{Autarky}},\ w)\right].$$

This explains why producers' surplus must increase for both countries for the case of equal variances. Now, suppose that, holding σ^* fixed, σ is increased from the initial value of $\sigma = \sigma^*$. It is easy to check that the above inequality will continue to hold for the Home country, which has the higher variance of the cost shock. This implies that Home country producers will be better off in the switch from autarky to trade.

Consider consumers' surplus next. Given w and w^*, the consumers' surplus under trade is

$$CS|_{\text{Trade}} = \frac{1}{2b}(a-p)^2.$$

Substituting for p, we get the expected consumers' surplus to be

$$E(CS|_{\text{Trade}}) = E(CS|_{\text{Autarky}}) - \frac{bn^2}{8(\beta+bn)^2}[3\sigma^2 - \sigma^{*2} - 2\rho\sigma\sigma^*). \tag{5}$$

Again, for the case of equal variances and $\rho = 1$, we do not expect any change in the expected consumers' surplus in either country, since trade keeps both the variance of price and the covariance of price with the cost shock unaffected. This is readily confirmed from equation (5). For the general case, we can draw the following conclusion about consumers' surplus using equation (5) (the proof is developed in Appendix A.2).

Proposition 2.

1. *The loss in expected consumers' surplus under trade is decreasing in ρ for both countries.*
2. *If σ is sufficiently close to σ^*, that is, if $\frac{\sqrt{\rho^2+3}+\rho}{3} < \frac{\sigma}{\sigma^*} < \sqrt{\rho^2+3}-\rho$, then, for both countries, the expected consumers' surplus is lower under trade than under autarky.*[12] *Otherwise, under trade, the expected consumers' surplus is lower for the country with the higher variance of the cost shock, and higher for the country with the lower variance of the cost shock.*[13]
3. *If $\sigma = \sigma^* = \bar{\sigma}$ and $\rho < 1$, the loss in expected consumers' surplus under trade is increasing in $\bar{\sigma}$.*

[12]Note that $\frac{\sqrt{\rho^2+3}+\rho}{3} \leq 1$, $\sqrt{\rho^2+3}-\rho \geq 1$ and $\left(\frac{\sqrt{\rho^2+3}+\rho}{3}\right)\left(\sqrt{\rho^2+3}-\rho\right) = 1$.

[13]To be precise, when $\frac{\sigma}{\sigma^*} \leq \frac{\sqrt{\rho^2+3}+\rho}{3} \leq 1$, the expected consumers' surplus under trade is higher for the Home country and lower for the Foreign country, and the situation reverses when $\frac{\sigma}{\sigma^*} \geq \sqrt{\rho^2+3}-\rho \geq 1$.

Notice that the gap between the lower and upper bounds on $\frac{\sigma}{\sigma^*}$ in Proposition 2.2 is decreasing in ρ and is zero for $\rho = 1$. In other words, as the correlation between the shocks increases, it becomes more likely that at least one of the countries' consumers will benefit from trade. However, it is never the case that consumers of both countries will benefit, that is, even in the limit, when $\rho = 1$, consumers of the country with the higher variance of the cost shock are worse off with trade.

The effect of trade on consumers' surplus is relatively straightforward to understand. We know from equation (1) that, expected price remaining the same, consumers' surplus increases with the variance of price. With trade the variance of price for the country with higher variance of the cost shock clearly goes down, and hence the consumers of this country are worse off with trade.[14] However, the variance of price for the country with lower variance under autarky may increase, especially if the correlation between the shocks is high, so that consumers in this country may be better off after trade. Recall that the gap between the bounds on $\frac{\sigma}{\sigma^*}$ in Proposition 2.2 tends to zero as ρ tends to 1, so that consumers in one of the countries will be better off in the limit if the variances are unequal.

Finally, consider social welfare. The expected social welfare under trade is

$$\begin{aligned} E(SW|_{\text{Trade}}) &= nE(\pi|_{\text{Trade}}) + E(CS|_{\text{Trade}}) \\ &= E(SW|_{\text{Autarky}}) + \frac{bn^2}{8\beta(\beta + bn)}[\sigma^2 + \sigma^{*2} - 2\rho\sigma\sigma^*]. \end{aligned} \tag{6}$$

Since we saw above that for the case of equal variances and $\rho = 1$, neither the expected producers' surplus nor the expected consumers' surplus change with trade, social welfare also remains unaffected, as is easily confirmed from equation (6). Proposition 3 summarizes the results for the general case.

Proposition 3.

1. *Social welfare is higher under trade than autarky for both the Home and the Foreign country for any value of the parameters σ, σ^** or *ρ, and the increase in social welfare under trade is decreasing in the correlation coefficient ρ for both countries.*
2. *When $\sigma = \sigma^* = \bar{\sigma}$ and $\rho < 1$, the gain in social welfare is increasing in $\bar{\sigma}$.*

[14] Even in the case of equal variances, as demonstrated earlier, trade decreases price variability, and hence results in a lower consumer surplus.

For the case of identical variances and $\rho < 1$, we know in Propositions 1 and 2 that producers in both countries are better off with trade, while consumers are worse off. Proposition 3, however, says that social welfare improves in both countries. This perhaps best highlights one major difference of our analysis from traditional trade theory, where, under perfect competition, the gains from trade are driven primarily by gains to consumers. In our setting, gains from trade can accrue to the producers and outweigh the loss to consumers. As equation (6) reveals, this is true irrespective of whether the variances are equal or not. Social welfare must be necessarily higher for both countries under trade.

The intuition for the results on social welfare can be developed in terms of the familiar "welfare triangles", as we now show. In Figure 1 we consider the situation from the home country's perspective when the effect of trade is to reduce the variance of price. We assume linear demand and supply curves. There are two possible realizations of the cost shock, assumed equally likely. The two positions of the supply curve labelled DU and FT correspond, respectively, to realizations of the positive and negative cost shock in the home country. We assume that the cost shocks in the home and the foreign country are perfectly negatively correlated, so that under trade, the home country producers and consumers face a constant price, given by the height OP.

The expected social surplus under autarky is given by

$$\frac{1}{2}(\text{Area DUC} + \text{Area FTC}).$$

The expected social surplus under trade is given by

$$\frac{1}{2}(\text{Area PRC} + \text{Area DQP}) + \frac{1}{2}(\text{Area PRC} + \text{Area FSP}).$$

From Figure 1, it is clear that the gain in social welfare from trade is positive, and given by

$$\frac{1}{2}(\text{Area QUR}) + \frac{1}{2}(\text{Area RTS}) = \text{Area QUR}.$$

It is also easy to check that the expected consumers' surplus is lower, and the expected producers' surplus is higher, under trade.[15]

[15] While in Figure 1 we consider the extreme case in which trade eliminates all variability in the product price, the impact of any reduction in variability is similar.

In Figure 2, we consider a situation in which the effect of trade is to increase the variance of price. Under autarky, the home country enjoys a stable price (there are no cost shocks) given by height OP. Trade introduces variability of price, since the other country is assumed to experience cost shocks. Expected social surplus under autarky is given by Area FRC. When trade opens up, the expected social surplus is

$$\frac{1}{2}(\text{Area DUC} + \text{Area FSD}) + \frac{1}{2}(\text{Area EWC+ Area FTE}).$$

It is easily checked that the social welfare under trade exceeds that under autarky by

$$\frac{1}{2}(\text{Area RSU}) + \frac{1}{2}\ (\text{Area TWR}) = \text{Area RSU}.$$

Thus, irrespective of whether trade increases or decreases the variability of price for the home country, social welfare increases.

4. Cournot Competition

In this section, we assume that the firms are Cournot competitors. A substantial literature has been developed to address the issues of trade, gains from trade and optimal trade policies when firms operate under strategic environments.[16] But whether individual rival firms from two separate countries themselves benefit from a move from autarky to free trade has not received much attention until recently. Anderson, Donsimoni and Gabszewicz (1989) consider a deterministic environment and show that producers' surplus in oligopolistic autarkic industries would be lower under trade due to the "market expansion effect". Roughly, trade or market expansion causes firms to expand output because with integrated markets, the demand curve facing the firms is flatter when they can serve both markets, thereby raising marginal revenue at a given (symmetric) level of output.[17] Our framework differs in that we have uncertainty affecting the marginal cost of production. The analysis of the previous section (the "variance-covariance effect" in particular) suggests, however, that there should be some offsetting benefits of trade. This is exactly what we find. In a relatively recent contribution Moner-Colonques (1998) addresses the same

[16] See, for example, Brander and Krugman (1983), Brander and Spencer (1985), Eaton and Grossman(1986), Markusen and Venables (1988), Qiu (1994) and Brainard and Martimort (1997), to mention just a few.

[17] The Anderson, Donsimoni and Gabszewicz (1989) result also holds if the markets are segmented; however, the explanation for the result is slightly different in this case.

issue in a game of incomplete information about costs realizations of individual firms. We discuss below the differences of our findings with that of Moner-Colonques (1998). Unlike Anderson et al. (1989) or Moner-Colonques (1998), we also look at the effect of trade on consumers' surplus and social welfare.

The demand and cost structures are the same as in the basic model. To simplify calculations, we further assume that $b = 1$, and $\beta = 0$.[18]

Proceeding as in the last section, we first note that $E(p|_{\text{Autarky}}) = \dfrac{a + n\alpha}{n+1}$, whereas $E(p|_{\text{Trade}}) = \dfrac{a + 2n\alpha}{2n+1}$. Then

$$E(p|_{\text{Trade}}) - E(p|_{\text{Autarky}}) = \frac{n(\alpha - a)}{(n+1)(2n+1)} < 0, \text{ since } a > \alpha.$$

Thus, unlike price-taking behavior, expected price decreases (and hence output increases) due to a move from autarky to free trade under strategic behavior. This is the source of the "market expansion effect". Clearly, producers are worse-off due to this effect whereas the consumers are better-off.

4.1. Do Firms Benefit From Trade?

In this subsection we address the question whether individual rival firms from two separate countries themselves benefit from a move from autarky to free trade. Using equation (3) (and noting that $b = 1$, and $\beta = 0$) we can decompose the gains from trade for individual firms as follows:

$$\begin{aligned}
&E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}}) \\
&= \left[E(p|_{\text{Trade}})\left(E(p|_{\text{Trade}}) - 2\alpha\right) - E(p|_{\text{Autarky}})\left(E(p|_{\text{Autarky}}) - 2\alpha\right)\right] \\
&+ \left[\{Var(p|_{\text{Trade}}) - 2Cov(p|_{\text{Trade}}, w)\} - \left\{Var(p|_{\text{Autarky}}) - 2Cov(p|_{\text{Autarky}}, w)\right\}\right].
\end{aligned}$$

The decomposition shows the two offsetting effects of free trade at work – the first term on the right-hand-side is the "market expansion effect" and the second one is the "variance-covariance effect". What happens to the gains from trade for individual firms depend on

[18]Note that we cannot assume $\beta = 0$ when the firms are price takers. The expected profits of the firms are then zero under all circumstances. But this problem does not arise when the firms are Cournot competitors.

the relative magnitudes of these two effects. Using the specifications of the model we can derive

$$
\begin{aligned}
& E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}}) \\
&= -\frac{(2n^2-1)(a-\alpha)^2}{(2n+1)^2(n+1)^2} \qquad (7) \\
&+ \frac{\sigma^2(2n^4+8n^3+8n^2+4n+1) + 2\sigma^{*2}n^2(n+1)^2 - 4\rho\sigma\sigma^* n(n+1)^3}{(2n+1)^2(n+1)^2}.
\end{aligned}
$$

It is easy to identify that the first term involving no uncertainty parameters captures the "market expansion effect" while the second term is the "variance-covariance effect". Note that the "market expansion effect" is negative, that is, individual firm's expected profit decreases under trade. Anderson et al. (1989) arrives at a similar conclusion under a deterministic environment. But, under cost uncertainty, we have the additional effect – the "variance-covariance effect" – which can be positive and can dominate the negative "market expansion effect".

Since we are interested in finding conditions when firms of *both* the Home and Foreign countries benefit from trade, let us express their gains in expected producers' surplus in the following way:

$$
\begin{aligned}
& (2n+1)^2(n+1)^2\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]_{\text{Home}} \qquad (8a) \\
&= -(2n^2-1)(a-\alpha)^2 \\
&+\sigma^{*2}\left[(2n^4+8n^3+8n^2+4n+1)\left(\frac{\sigma}{\sigma^*}\right)^2 - 4\rho n(n+1)^3\left(\frac{\sigma}{\sigma^*}\right) + 2n^2(n+1)^2\right],
\end{aligned}
$$

and

$$
\begin{aligned}
& (2n+1)^2(n+1)^2\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]_{\text{Foreign}} \qquad (8b) \\
&= -(2n^2-1)(a-\alpha)^2 \\
&+\sigma^{*2}\left[2n^2(n+1)^2\left(\frac{\sigma}{\sigma^*}\right)^2 - 4\rho n(n+1)^3\left(\frac{\sigma}{\sigma^*}\right) + (2n^4+8n^3+8n^2+4n+1)\right].
\end{aligned}
$$

Let us denote the coefficients of σ^{*2} in (8a) and (8b) by $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ respectively. For $\rho < 0$, clearly, both $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ are strictly positive . For $\rho > 0$, it is shown in Appendix A.3 that both $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ are strictly positive if $\rho^2 < 1 - \frac{(2n+1)^2}{2(n+1)^4}$, and when $\rho^2 \geq 1 - \frac{(2n+1)^2}{2(n+1)^4}$, they are both strictly positive when σ is sufficiently different from σ^* (either $\frac{\sigma}{\sigma^*} < z_1$ or $\frac{\sigma}{\sigma^*} > z_4$).[19] For a given $\frac{\sigma}{\sigma^*}$, define, $\underline{V}\left(\frac{\sigma}{\sigma^*}\right) = \min\left\{V^H\left(\frac{\sigma}{\sigma^*}\right), V^F\left(\frac{\sigma}{\sigma^*}\right)\right\}$. If $\underline{V}\left(\frac{\sigma}{\sigma^*}\right) > 0$ and the amount of uncertainty is sufficiently high such that $\sigma^{*2} > \frac{(2n^2-1)(a-\alpha)^2}{\underline{V}\left(\frac{\sigma}{\sigma^*}\right)}$, then firms of both countries benefit from trade. Proposition 4 summarizes this conclusion:

Proposition 4. *When the firms are Cournot competitors, the expected producers' surplus of both Home and Foreign firms are higher under trade than under autarky if either* (i) $\rho < 0$ *and, for a given* $\frac{\sigma}{\sigma^*}$, $\sigma^{*2} > \frac{(2n^2-1)(a-\alpha)^2}{\underline{V}\left(\frac{\sigma}{\sigma^*}\right)}$, (ii) $\rho > 0$, $\rho^2 < 1 - \frac{(2n+1)^2}{2(n+1)^4}$ *and, for a given* $\frac{\sigma}{\sigma^*}$, $\sigma^{*2} > \frac{(2n^2-1)(a-\alpha)^2}{\underline{V}\left(\frac{\sigma}{\sigma^*}\right)}$, *or* (iii) $\rho > 0$, $\rho^2 \geq 1 - \frac{(2n+1)^2}{2(n+1)^4}$, σ *is sufficiently different from* σ^* (either $\frac{\sigma}{\sigma^*} < z_1$ or $\frac{\sigma}{\sigma^*} > z_4$), *and, for a given* $\frac{\sigma}{\sigma^*}$, $\sigma^{*2} > \frac{(2n^2-1)(a-\alpha)^2}{\underline{V}\left(\frac{\sigma}{\sigma^*}\right)}$.

Proposition 4 is analogous to Proposition 1.3 – 1.5. Because of the pro-competitive effect, for producers' surplus to increase, we need the variance of the cost shocks to be sufficiently high and the correlation to be sufficiently small.[20]

Notice that the upper bound on ρ^2 in part (ii) of the Proposition is increasing in n, and for $n = 1$, has a value of 0.718 (i.e., the condition holds for $|\rho| < 0.8488$). However, since the numerator and denominator of the lower bound on σ^2 both depend on n, we cannot immediately conclude that the condition is more likely to hold if n is larger.

We can get more precise conditions if we further assume that the variances of the cost

[19] z_1 and z_4 are defined in Appendix A.3.

[20] Another difference is that for the price-taking case, if the variances of the cost shocks are sufficiently close, both countries benefit from trade. This is true in the Cournot case only if the correlation coefficient is less than the upper bound given in part (ii) of Proposition 4.

shocks are identical: $\sigma = \sigma^* = \bar{\sigma}$. Then the expected gain from trade becomes

$$\begin{aligned} &\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]\Big|_{\sigma=\sigma^*=\bar{\sigma}} \\ &= \frac{-(2n^2-1)(a-\alpha)^2 + \bar{\sigma}^2\left[4n(n+1)^3(1-\rho) - (2n^2-1)\right]}{(2n+1)^2(n+1)^2}. \end{aligned} \tag{9}$$

The following proposition now follows from equation (9).

Proposition 5. *When the firms are Cournot competitors and $\sigma = \sigma^* = \bar{\sigma}$, the expected producers' surplus of both Home and Foreign firms are higher under trade than under autarky if and only if* (i) $\rho < 1 - \dfrac{2n^2-1}{4n(n+1)^3}$ *and* (ii) $\bar{\sigma}^2 > \dfrac{(2n^2-1)(a-\alpha)^2}{4n(n+1)^3(1-\rho) - (2n^2-1)}$.

It turns out that the expressions in the right hand side of Proposition 5 (i) decreases slightly as n goes from $n = 1$ to $n = 2$, and then increase monotonically with n. Exactly the opposite is true for the expression in the right hand side of Proposition 5 (ii), which increases from $n = 1$ to $n = 2$, and then decreases monotonically in n. Thus, in the symmetric case, as the number of firms in each industry increases beyond the duopoly case of $n = 2$, it is more likely that firms in both countries will benefit from the switch to trade.

In Proposition 5, we need the correlation coefficient to be sufficiently smaller than 1 and the variances of the cost shocks to be sufficiently high for the gain to the producers from the "variance-covariance effect" to dominate the loss from the "market expansion effect". Our earlier discussion on how the "variance-covariance effect" affects expected profit is useful in understanding the various effects at work. In the *price-taking case*, note that, for $\rho < 1$, the net gains in expected profit due to the "variance-covariance effect" is positive and increasing in $\bar{\sigma}^2$.[21] Thus, the gains from trade due to the "variance-covariance effect" exist if and only if the correlation is less than perfect, and increase in the common variance of the cost shock. In the Cournot case considered in Proposition 5, the

[21] Recall that when firms are price takers, we have:

$$\begin{aligned} &[Var(p|_{\text{Trade}}) - 2Cov(p|_{\text{Trade}}\,,\ w)] - \left[Var(p|_{\text{Autarky}}) - 2Cov(p|_{\text{Autarky}}\,,\ w)\right] \\ = &-\frac{1}{2}\left(\frac{bn}{\beta+bn}\right)^2 \bar{\sigma}^2(1-\rho) + \left(\frac{bn}{\beta+bn}\right)\bar{\sigma}^2(1-\rho). \end{aligned}$$

conditions necessary for gains from trade are similar to the price -taking case. So long as the correlation is not too close to one, the gain from trade from the "variance-covariance effect" can outweigh the loss due to the "market expansion effect" if the variance of the shock is sufficiently high.

It is interesting to compare our findings with that of Moner-Colonques (1998). He shows that in the presence of private cost information, the expected profit of an oligopolistic firm is higher under free trade than under autarky when there exists a sufficiently large amount of uncertainty and a certain degree of firms' heterogeneity. We do not need any asymmetry of information for our result. We also need uncertainty to be sufficiently large, but that is to strengthen the "variance-covariance effect", which is quite intuitive. Interestingly, in Moner-Colonques' analysis, the firms of at least one country prefer to operate under autarky rather than under free trade, for the particular case of symmetry both in demand and industry sizes, whereas this symmetric case is precisely what we have considered in Proposition 5.

4.2. Gains in Consumers' Surplus and Social Welfare

Now we analyze the effect of trade on consumers' surplus and social welfare. Consider consumers' surplus first. From equation (1) it is clear that gains in consumers' surplus depend on the relative magnitudes of the "market expansion effect" and the effect of price variability on consumers' surplus. Using the model specifications we get the expression for the gain in consumers' surplus as

$$\begin{aligned} &E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}}) \\ &= \frac{n^2(4n+3)(a-\alpha)^2}{2(2n+1)^2(n+1)^2} \\ &+ \frac{n^2\left[(n+1)^2(\sigma^{*2}+2\rho\sigma\sigma^*) - n(3n+2)\sigma^2\right]}{2(2n+1)^2(n+1)^2}. \end{aligned} \tag{10}$$

The first term represents the effect of market expansion and the second term captures the impact of price variability. Not surprisingly, market expansion and the associated increase in production tends to increase the consumers' surplus. However, trade may reduce the variance of price and the offsetting effect of lower price variability tends to lower consumers' surplus. Proposition 6 summarizes the results for gains in consumers' surplus that follow from equation (10) and is proved in Appendix A.4.

Proposition 6. *When the firms are Cournot competitors,*

1. *The expected consumers' surplus is lower under trade than under autarky for both countries if and only if* $\rho < \dfrac{2n^2-1}{2(n+1)^2}$, σ *is sufficiently close to* σ^*,[22] *and, for a given* $\dfrac{\sigma}{\sigma^*}$, σ^{*2} *is sufficiently high.*[23]

2. *For any value of* ρ, *if* σ *is sufficiently different from* σ^*,[24] *the expected consumers' surplus under trade is* (i) *higher for the country with the lower variance of the cost shock and* (ii) *lower for the country with the higher variance of the cost shock if, for a given* $\dfrac{\sigma}{\sigma^*}$, σ^{*2} *is sufficiently high.*[25]

These results mirror the results in Proposition 2.2 for the price-taking case. One difference is that, because of the pro-competitive effect, we need the variance of the costs shocks to be sufficiently high for consumer welfare to decrease following trade.

Several comments are in order. First, from Proposition 6.1, the value of ρ necessary for consumers' surplus to increase following trade in at least one country is not high for the monopoly or the duopoly cases (the upper bound is 0.389 for $n = 2$). However, when the correlation is close to zero (or negative) , and the variances are similar in magnitude and sufficiently high, consumers' surplus will decrease in both countries with the switch from autarky to trade.[26] Finally, holding the variance of one of the countries (say the Home country) unchanged, as the variance of the Foreign country increases sufficiently, consumers' surplus will increase in the home country but decrease in the foreign country.

---

[22] Specifically, $\frac{\sigma}{\sigma^*}$ need to lie in an interval (s, s^*), where s and s^* are defined in Appendix A.4, and it is shown that $s < 1$ and $s^* > 1$ when $\rho < \dfrac{2n^2-1}{2(n+1)^2}$.

[23] Specifically, $\sigma^{*2} > \dfrac{(4n+3)(a-\alpha)^2}{\underline{S}\left(\frac{\sigma}{\sigma^*}\right)}$, where $\underline{S}(\cdot)$ is defined in Appendix A.4.

[24] That is, if $\dfrac{\sigma}{\sigma^*} \notin [\underline{s}, \overline{s}]$, where $\underline{s}$, $\overline{s}$ are defined in Appendix A.4, and $\underline{s} \leq 1$ and $\overline{s} \geq 1$.

[25] Specifically, $\sigma^{*2} > \dfrac{(4n+3)(a-\alpha)^2}{\left|S^i\left(\frac{\sigma}{\sigma^*}\right)\right|}$, where $S^i\left(\frac{\sigma}{\sigma^*}\right)$ is defined in Appendix A.4, i being the index for the country with the higher variance of the cost shock.

[26] Only if the correlation is perfect ($\rho = 1$) and the variance of the shock the same will the variability of the price remain unchanged after trade for both countries. Otherwise, price variability will decline (when the variances are the same for both countries), and consumers may be worse off.

Once again, the conditions can be made more precise when $\sigma = \sigma^* = \bar{\sigma}$. In that case, the gain in consumers' surplus from trade is

$$\begin{aligned} &\left[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\right]\Big|_{\sigma=\sigma^*=\bar{\sigma}} \\ &= \frac{n^2\left[(4n+3)(a-\alpha)^2 - \bar{\sigma}^2\left\{n(3n+2)-(n+1)^2(1+2\rho)\right\}\right]}{2(2n+1)^2(n+1)^2}. \end{aligned} \tag{11}$$

Now we can draw the following conclusion that follows from equation (11).

Proposition 7: *When the firms are Cournot competitors and $\sigma = \sigma^* = \bar{\sigma}$, the expected consumers' surplus is lower under trade than under autarky if and only if $\rho < \dfrac{2n^2-1}{2(n+1)^2}$ and $\bar{\sigma}^2 > \dfrac{(4n+3)(a-\alpha)^2}{n(3n+2)-(n+1)^2(1+2\rho)}$.*

Finally, we compare social welfare under autarky and trade. For a wide range of values for the variance of the cost shock, once again, remarkably, social welfare is higher in both countries under trade than under autarky, for all values of the correlation coefficient. But, interestingly, we can pin down the precise conditions under which social welfare may become lower under trade than under autarky.

The expected social welfare gain under trade is

$$\begin{aligned} &E(SW|_{\text{Trade}}) - E(SW|_{\text{Autarky}}) \\ &= \frac{n}{2(n+1)^2(2n+1)^2}\left[(a-\alpha)^2(3n+2) + \sigma^2\left(4n^4+13n^3+14n^2+8n+2\right)\right. \\ &\qquad \left. -2n\rho\sigma\sigma^*(4n+3)(n+1)^2 + n\sigma^{*2}(4n+1)(n+1)^2\right]. \end{aligned} \tag{12}$$

Like the expected producers' surplus and consumers' surplus, the gain in social welfare also is affected by the two effects of market expansion and price variability. It is clear from equation (12) that the net effect of market expansion on social welfare is positive. For a wide range of parameter values, the impact of trade on the variability of price and its covariability with the cost-shock in fact reinforces this positive effect. When n is large, given our results from the price-taking case, this is exactly what we would expect. In fact, for $\rho = 1$, the expression representing the "variance-covariance effect" in equation (12),

$\sigma^2\left[\dfrac{(4n^4+13n^3+14n^2+8n+2)}{n(4n+1)(n+1)^2} - 2\left(\dfrac{\sigma^*}{\sigma}\right)\dfrac{(4n+3)}{4n+1} + \left(\dfrac{\sigma^*}{\sigma}\right)^2\right]$, converges to a perfect square (for given $\frac{\sigma^*}{\sigma} \neq 1$) as n approaches infinity. This parallels the results of Proposition 3 (see equation (6)). Nonetheless, for finite n, it is possible for the expression to be negative.[27] If the correlation coefficient is sufficiently close to 1, and the variance of the cost shock is sufficiently high for the home country, the loss in social welfare due to the impact of trade on the variability of price and its covariability with the cost shock may outweigh the gain from the market expansion effect. However, as the next Proposition shows, the range of parameter values for which this can happen is still quite small.

Proposition 8.

1. *The expected social welfare gain is monotonically decreasing in ρ for both countries.*
2. *For any value of ρ, social welfare under trade is higher than under autarky for the country with the higher variance of the cost shock.*
3. *For any value of ρ, social welfare under trade is higher for both countries if either $\frac{\sigma}{\sigma^*} \leq \frac{4n+1}{4n+5}$, or $\frac{\sigma}{\sigma^*} \geq \frac{4n+5}{4n+1}$.*
4. *When $\rho^2 < 1 - \frac{(n-2)(2n+1)^2}{n(4n+3)^2(n+1)^2}$, the expected social welfare under trade is higher for both countries.*
5. *When $\rho^2 \geq 1 - \frac{(n-2)(2n+1)^2}{n(4n+3)^2(n+1)^2}$, the expected social welfare under trade is higher for both countries if either σ is sufficiently close to σ^* $\left(t_2 \leq \frac{\sigma}{\sigma^*} \leq t_3\right)$ or σ is sufficiently different from σ^* $\left(\text{either } \frac{\sigma}{\sigma^*} \leq t_1, \text{ or } \frac{\sigma}{\sigma^*} \geq t_4\right)$.[28] Otherwise, the expected social welfare under trade is higher for the country with the higher variance of the cost shock, and it is higher for the country with the lower variance of the cost shock if, for a given $\frac{\sigma}{\sigma^*}$, $\sigma^{*2} < \frac{(3n+2)(a-\alpha)^2}{\left|W^i\left(\frac{\sigma}{\sigma^*}\right)\right|}$, where i is the index for the country with the lower variance of the cost shock.*[29]

[27] To see this, note that for given n, the value of $\frac{\sigma^*}{\sigma}$ that minimizes the expression is $\frac{\sigma^*}{\sigma} = \frac{(4n+3)}{4n+1}$. Substituting this value into the above expression, one can check that the expression is negative for $n > 2$, although it rapidly approaches zero as n increases. In other words, if the correlation coefficient is sufficiently close to 1, for every $n > 2$, one can find $\frac{\sigma^*}{\sigma}$ such that the expression is negative.

[28] t_1, t_2, t_3 and t_4 are defined in Appendix A.5. Note that $\frac{4n+1}{4n+5} \leq t_1 < t_2 \leq 1 \leq t_3 < t_4 \leq \frac{4n+5}{4n+1}$.

[29] For the definition of $W^i(\cdot)$, $i = H, F$, see Appendix A.5.

6. **Immiserizing Trade:** *Social welfare under trade is lower than that under autarky* (i) *for the Home country if and only if* $\rho^2 \geq 1 - \frac{(n-2)(2n+1)^2}{n(4n+3)^2(n+1)^2}$, $t_1 \leq \frac{\sigma}{\sigma^*} \leq t_2$, *and, for a given* $\frac{\sigma}{\sigma^*}$, $\sigma^{*2} > \frac{(3n+2)(a-\alpha)^2}{\left|W^H\left(\frac{\sigma}{\sigma^*}\right)\right|}$, *and* (ii) *for the Foreign country if and only if* $\rho^2 \geq 1 - \frac{(n-2)(2n+1)^2}{n(4n+3)^2(n+1)^2}$, $t_3 \leq \frac{\sigma}{\sigma^*} \leq t_4$, *and, for a given* $\frac{\sigma}{\sigma^*}$, $\sigma^{*2} > \frac{(3n+2)(a-\alpha)^2}{\left|W^F\left(\frac{\sigma}{\sigma^*}\right)\right|}$.

Unlike Proposition 3, social welfare is not always higher for both countries under trade than under autarky when there is Cournot competition. However, it is still higher for both countries for a wide range of parameter values. Notice that unless the shocks are highly correlated, the condition in Proposition 8.6 is not going to be met. The minimum value of the upper bound occurs for $n = 3$ and is 0.995. Even when this condition is met, for social welfare to decrease, the ratio of the variance of the shocks must not be very close to or very far away from 1, and the variances must be sufficiently high. It is also clear from the conditions on t_1, t_2, t_3, t_4 and Proposition 8.6 that both countries cannot be worse off from trade. The underlying reason why social welfare increases in the Cournot case except under very tight parameter configurations is much the same as in the price-taking case where it always increases. Imperfect competition may prevent arbitrage opportunities to be completely exploited; nonetheless, at least in our linear model, the gains from trade seem surprisingly robust.

5. Conclusion

In this paper, we show that trade can affect the welfare of countries in the presence of arbitrage opportunities as it affects the exposure of the countries to uncertainty. Producers' surplus is affected due to the "variance-covariance" effect. Consumers are also affected as the variability of product prices changes. Depending on the variances of the shocks, the correlation between the shocks and the number of firms, producers' and consumers' surplus in a given country can be either higher or lower with trade than under autarky. However, social welfare is higher in both countries under a surprisingly robust set of conditions, both when the firms are price-takers or Cournot competitors.

6. Appendix

A.1. Proof of Proposition 1.

Consider a Home country firm first. Equation (4) can be rewritten as

$$\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]_{\text{Home}}$$
$$= \frac{bn\sigma^{*2}}{8\beta(\beta+bn)^2}\left[(4\beta+bn)\left(\frac{\sigma}{\sigma^*}\right)^2 - (4\beta+2bn)\rho\left(\frac{\sigma}{\sigma^*}\right) + bn\right].$$

Consider the quadratic expression

$$(4\beta+bn)\left(\frac{\sigma}{\sigma^*}\right)^2 - (4\beta+2bn)\rho\left(\frac{\sigma}{\sigma^*}\right) + bn. \tag{A.1}$$

This expression is strictly convex in $\frac{\sigma}{\sigma^*}$ and its *minimum* value is $\frac{4bn(4\beta+bn)-(4\beta+2bn)^2\rho^2}{4(4\beta+bn)}$. It follows that, for a Home country firm, $E(\pi|_{\text{Trade}}) > E(\pi|_{\text{Autarky}})$ when $\rho^2 < \frac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$.

When $\rho^2 \geq \frac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$, (A.1) can be expressed as

$$(4\beta+bn)\left(\frac{\sigma}{\sigma^*}\right)^2 - (4\beta+2bn)\rho\left(\frac{\sigma}{\sigma^*}\right) + bn = (4\beta+bn)\left(\frac{\sigma}{\sigma^*}-x_1\right)\left(\frac{\sigma}{\sigma^*}-x_2\right),$$

where $x_1 = \frac{(4\beta+2bn)\rho - \sqrt{(4\beta+2bn)^2\rho^2 - 4bn(4\beta+bn)}}{2(4\beta+bn)}$ and $x_2 = \frac{(4\beta+2bn)\rho + \sqrt{(4\beta+2bn)^2\rho^2 - 4bn(4\beta+bn)}}{2(4\beta+bn)}$. It is easy to check that $\frac{bn}{4\beta+bn} \leq x_1 < x_2 \leq 1$. Now it follows that, if $\rho^2 \geq \frac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$,

$$\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]_{\text{Home}} \begin{cases} \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \leq x_1, \\ < 0, \text{ when } x_1 < \frac{\sigma}{\sigma^*} < x_2, \\ \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \geq x_2. \end{cases}$$

Now consider a Foreign firm. By symmetry, it follows from equation (4) that

$$\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]_{\text{Foreign}}$$

$$\frac{bn}{8\beta(\beta+bn)^2}[(4\beta+bn)\sigma^{*2} + bn\sigma^2 - (4\beta+2bn)\rho\sigma\sigma^*]$$

$$= \frac{bn\sigma^{*2}}{8\beta(\beta+bn)^2}\left[bn\left(\frac{\sigma}{\sigma^*}\right)^2 - (4\beta+2bn)\rho\left(\frac{\sigma}{\sigma^*}\right) + (4\beta+bn)\right].$$

Consider the quadratic expression

$$bn\left(\frac{\sigma}{\sigma^*}\right)^2 - (4\beta+2bn)\rho\left(\frac{\sigma}{\sigma^*}\right) + (4\beta+bn). \tag{A.2}$$

This expression is also strictly convex in $\frac{\sigma}{\sigma^*}$ with a *minimum* value of $\frac{4bn(4\beta+bn) - (4\beta+2bn)^2\rho^2}{4bn}$. Thus, for a Foreign firm also $E(\pi|_{\text{Trade}}) > E(\pi|_{\text{Autarky}})$ when $\rho^2 < \frac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$.

When $\rho^2 \geq \frac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$, (A.2) can be expressed as

$$bn\left(\frac{\sigma}{\sigma^*}\right)^2 - (4\beta+2bn)\rho\left(\frac{\sigma}{\sigma^*}\right) + (4\beta+bn) = bn\left(\frac{\sigma}{\sigma^*} - x_3\right)\left(\frac{\sigma}{\sigma^*} - x_4\right),$$

where $x_3 = \frac{(4\beta+2bn)\rho - \sqrt{(4\beta+2bn)^2\rho^2 - 4bn(4\beta+bn)}}{2bn}$ and $x_4 = \frac{(4\beta+2bn)\rho + \sqrt{(4\beta+2bn)^2\rho^2 - 4bn(4\beta+bn)}}{2bn}$. It is easy to check that $1 \leq x_3 < x_4 \leq \frac{4\beta+bn}{bn}$.[30] Hence it follows that, if $\rho^2 \geq \frac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$,

$$\left[E(\pi|_{\text{Trade}}) - E(\pi|_{\text{Autarky}})\right]_{\text{Foreign}} \begin{cases} \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \leq x_3, \\ < 0, \text{ when } x_3 < \frac{\sigma}{\sigma^*} < x_4, \\ \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \geq x_4. \end{cases}$$

[30] Also, note that $x_1x_4 = 1$, and $x_2x_3 = 1$.

Thus, when $\rho^2 < \dfrac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$, the expected producers' surplus is higher under trade than under autarky for firms in both countries (Proposition 1.4). When $\rho^2 \geq \dfrac{4bn(4\beta+bn)}{(4\beta+2bn)^2}$, the expected producers' surplus is higher under trade than under autarky for firms in both countries if either σ is sufficiently close to σ^* (that is, $x_2 \leq \dfrac{\sigma}{\sigma^*} \leq x_3$) or σ is sufficiently different from σ^* (that is, either $\dfrac{\sigma}{\sigma^*} \leq x_1$, or $\dfrac{\sigma}{\sigma^*} \geq x_4$); otherwise, when $x_1 < \dfrac{\sigma}{\sigma^*} < x_2$, Home firms suffer a loss in producers' surplus while Foreign firms enjoy a gain, and the situation reverses when $x_3 < \dfrac{\sigma}{\sigma^*} < x_4$ (Proposition 1.5).[31]

Proposition 1.3 now follows from Propositions 1.4 and 1.5. ■

A.2. Proof of Proposition 2.

From equation (5) it follows that

$$\begin{aligned}
&\Big[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\Big]_{\text{Home}} \\
&= -\frac{bn^2}{8(\beta+bn)^2}[3\sigma^2 - \sigma^{*2} - 2\rho\sigma\sigma^*) \\
&= -\frac{bn^2\sigma^{*2}}{8(\beta+bn)^2}\left[3\left(\frac{\sigma}{\sigma^*}\right)^2 - 2\rho\left(\frac{\sigma}{\sigma^*}\right) - 1\right] \\
&= -\frac{3bn^2\sigma^{*2}}{8(\beta+bn)^2}\left(\frac{\sigma}{\sigma^*} - y_1\right)\left(\frac{\sigma}{\sigma^*} - y_2\right),
\end{aligned}$$

where $y_1 = \dfrac{\rho - \sqrt{\rho^2+3}}{3}$ and $y_2 = \dfrac{\rho + \sqrt{\rho^2+3}}{3}$. Since $y_1 = \dfrac{\rho - \sqrt{\rho^2+3}}{3} < 0$ for $-1 \leq \rho \leq 1$, and $\dfrac{\sigma}{\sigma^*} \geq 0$, we can conclude that

$$\Big[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\Big]_{\text{Home}} \gtreqless 0 \text{ according as } \frac{\sigma}{\sigma^*} \lesseqgtr \frac{\rho + \sqrt{\rho^2+3}}{3}.$$

[31] Recall that $\dfrac{bn}{4\beta+bn} \leq x_1 < x_2 \leq 1 \leq x_3 < x_4 \leq \dfrac{4\beta+bn}{bn}$.

For the Foreign country we can similarly show that

$$\begin{aligned}
&\left[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\right]_{\text{Foreign}} \\
&= -\frac{bn^2}{8(\beta+bn)^2}[3\sigma^{*2} - \sigma^2 - 2\rho\sigma\sigma^*) \\
&= \frac{bn^2\sigma^{*2}}{8(\beta+bn)^2}\left[\left(\frac{\sigma}{\sigma^*}\right)^2 + 2\rho\left(\frac{\sigma}{\sigma^*}\right) - 3\right] \\
&= \frac{bn^2\sigma^{*2}}{8(\beta+bn)^2}\left(\frac{\sigma}{\sigma^*} - y_3\right)\left(\frac{\sigma}{\sigma^*} - y_4\right),
\end{aligned}$$

where $y_3 = -\rho - \sqrt{\rho^2+3}$, and $y_4 = -\rho + \sqrt{\rho^2+3}$. Since $y_3 = -\rho - \sqrt{\rho^2+3} < 0$ for $-1 \leq \rho \leq 1$, and $\frac{\sigma}{\sigma^*} \geq 0$, we can conclude that

$$\left[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\right]_{\text{Foreign}} \gtreqless 0 \text{ according as } \frac{\sigma}{\sigma^*} \gtreqless -\rho + \sqrt{\rho^2+3}.$$

Now Proposition 2.2 follows. ■

A.3. Proof of Proposition 4.

We have

$$V^H\left(\frac{\sigma}{\sigma^*}\right) = (2n^4+8n^3+8n^2+4n+1)\left(\frac{\sigma}{\sigma^*}\right)^2 - 4\rho n(n+1)^3\left(\frac{\sigma}{\sigma^*}\right) + 2n^2(n+1)^2, \text{ and}$$

$$V^F\left(\frac{\sigma}{\sigma^*}\right) = 2n^2(n+1)^2\left(\frac{\sigma}{\sigma^*}\right)^2 - 4\rho n(n+1)^3\left(\frac{\sigma}{\sigma^*}\right) + (2n^4+8n^3+8n^2+4n+1).$$

Both $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ are strictly convex in $\frac{\sigma}{\sigma^*}$, and their minimum values are

$$\min_{\left\{\frac{\sigma}{\sigma^*}\geq 0\right\}} V^H\left(\frac{\sigma}{\sigma^*}\right) = (2n^4+8n^3+8n^2+4n+1) - 2(n+1)^4\rho^2, \text{ and}$$

$$\min_{\left\{\frac{\sigma}{\sigma^*}\geq 0\right\}} V^F\left(\frac{\sigma}{\sigma^*}\right) = \frac{2n^2(n+1)^2\left[(2n^4+8n^3+8n^2+4n+1) - 2(n+1)^4\rho^2\right]}{2n^4+8n^3+8n^2+4n+1}.$$

Clearly, both $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ are strictly positive when

$\rho^2 < \frac{2n^4+8n^3+8n^2+4n+1}{2\left(n+1\right)^4} = 1 - \frac{\left(2n+1\right)^2}{2\left(n+1\right)^4}$.

When $\rho^2 \geq 1 - \frac{\left(2n+1\right)^2}{2\left(n+1\right)^4}$, proceeding as in subsection A.1, $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ can be written as

$$V^H\left(\frac{\sigma}{\sigma^*}\right) = (2n^4+8n^3+8n^2+4n+1)\left(\frac{\sigma}{\sigma^*}-z_1\right)\left(\frac{\sigma}{\sigma^*}-z_2\right), \text{ and}$$

$$V^F\left(\frac{\sigma}{\sigma^*}\right) = 2n^2(n+1)^2\left(\frac{\sigma}{\sigma^*}-z_3\right)\left(\frac{\sigma}{\sigma^*}-z_4\right),$$

where z_1 and z_2 $(z_1 < z_2)$ are the roots of the quadratic equation $V^H\left(\frac{\sigma}{\sigma^*}\right) = 0$, and z_3 and z_4 $(z_3 < z_4)$ are the roots of the quadratic equation $V^F\left(\frac{\sigma}{\sigma^*}\right) = 0$. It can be checked that $z_1 z_4 = 1$ and $z_2 z_3 = 1$, and $z_1 < z_3$ and $z_2 < z_4$. Now we can conclude that both $V^H\left(\frac{\sigma}{\sigma^*}\right)$ and $V^F\left(\frac{\sigma}{\sigma^*}\right)$ are strictly positive if either $\frac{\sigma}{\sigma^*} < z_1$ or $\frac{\sigma}{\sigma^*} > z_4$. ■

A.4. Proof of Proposition 6.

Using equation (10) we can express the gains in consumers' surplus for the Home and the Foreign country as follows:

$$\frac{2(2n+1)^2(n+1)^2}{n^2}\left[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\right]_{\text{Home}} \tag{A.3}$$

$$= (4n+3)(a-\alpha)^2 + \sigma^{*2}\left[-n(3n+2)\left(\frac{\sigma}{\sigma^*}\right)^2 + 2\rho(n+1)^2\left(\frac{\sigma}{\sigma^*}\right) + (n+1)^2\right], \text{ and}$$

$$\frac{2(2n+1)^2(n+1)^2}{n^2}\left[E(CS|_{\text{Trade}}) - E(CS|_{\text{Autarky}})\right]_{\text{Foreign}}$$

$$= (4n+3)(a-\alpha)^2 + \sigma^{*2}\left[(n+1)^2\left(\frac{\sigma}{\sigma^*}\right)^2 + 2\rho(n+1)^2\left(\frac{\sigma}{\sigma^*}\right) - n(3n+2)\right]. \tag{A.4}$$

Let us denote the coefficients of σ^{*2} in (A.3) and (A.4) by $S^H\left(\frac{\sigma}{\sigma^*}\right)$ and $S^F\left(\frac{\sigma}{\sigma^*}\right)$ respectively. Proceeding as in subsection A.2, we can show that

$$S^H\left(\frac{\sigma}{\sigma^*}\right) \gtreqqless 0 \text{ according as } \frac{\sigma}{\sigma^*} \lesseqqgtr \frac{2\rho\left(n+1\right)^2 + 2\sqrt{\left(n+1\right)^4\rho^2 + n\left(3n+2\right)\left(n+1\right)^2}}{2n\left(3n+2\right)} \equiv s, \text{ and}$$

$$S^F\left(\frac{\sigma}{\sigma^*}\right) \gtreqqless 0 \text{ according as } \frac{\sigma}{\sigma^*} \gtreqqless \frac{-2\rho(n+1)^2 + 2\sqrt{(n+1)^4\rho^2 + n(3n+2)(n+1)^2}}{2(n+1)^2} \equiv s^*.$$

Note that $ss^* = 1$. Also, it can be checked that (i) $s < 1$ and $s^* > 1$ when $\rho < \frac{2n^2-1}{2(n+1)^2}$, and (ii) $s \geq 1$ and $s^* \leq 1$ when $\rho \geq \frac{2n^2-1}{2(n+1)^2}$. For any value of ρ, define, $\underline{s} = \min\{s,\ s^*\}$, and $\overline{s} = \max\{s,\ s^*\}$. Also, for a given $\frac{\sigma}{\sigma^*}$, define, $\underline{S}\left(\frac{\sigma}{\sigma^*}\right) = \min\left\{\left|S^H\left(\frac{\sigma}{\sigma^*}\right)\right|,\ \left|S^F\left(\frac{\sigma}{\sigma^*}\right)\right|\right\}$. Now the conclusions in Proposition 6 follow.

A.5. Proof of Proposition 8.

Using equation (12) we can write the expressions for social welfare gains under trade for the Home and Foreign country as follows:

$$\begin{aligned}\frac{2(2n+1)^2(n+1)^2}{n}&\left[E(SW|_{\text{Trade}}) - E(SW|_{\text{Autarky}})\right]_{\text{Home}} = (3n+2)(a-\alpha)^2 \\ +\sigma^{*2}&\left[(4n^4+13n^3+14n^2+8n+2)\left(\frac{\sigma}{\sigma^*}\right)^2\right. \\ &\left.-2n\rho(4n+3)(n+1)^2\left(\frac{\sigma}{\sigma^*}\right) + n(4n+1)(n+1)^2\right], \text{ and}\end{aligned} \tag{A.5}$$

$$\begin{aligned}\frac{2(2n+1)^2(n+1)^2}{n}&\left[E(SW|_{\text{Trade}}) - E(SW|_{\text{Autarky}})\right]_{\text{Foreign}} = (3n+2)(a-\alpha)^2 \\ +\sigma^{*2}&\left[n(4n+1)(n+1)^2\left(\frac{\sigma}{\sigma^*}\right)^2 - 2n\rho(4n+3)(n+1)^2\left(\frac{\sigma}{\sigma^*}\right)\right. \\ &\left.+(4n^4+13n^3+14n^2+8n+2)\right].\end{aligned} \tag{A.6}$$

Let us denote the coefficients of σ^{*2} in (A.5) and (A.6) by $W^H\left(\frac{\sigma}{\sigma^*}\right)$ and $W^F\left(\frac{\sigma}{\sigma^*}\right)$ respectively. It can be checked that both $W^H\left(\frac{\sigma}{\sigma^*}\right)$ and $W^F\left(\frac{\sigma}{\sigma^*}\right)$ are strictly positive when $\rho^2 < 1 - \frac{(n-2)(2n+1)^2}{n(4n+3)^2(n+1)^2}$.

When $\rho^2 \geq 1 - \frac{(n-2)(2n+1)^2}{n(4n+3)^2(n+1)^2}$, $W^H\left(\frac{\sigma}{\sigma^*}\right)$ and $W^F\left(\frac{\sigma}{\sigma^*}\right)$ can be written as

$$W^H\left(\frac{\sigma}{\sigma^*}\right) = \left(4n^4+13n^3+14n^2+8n+2\right)\left(\frac{\sigma}{\sigma^*} - t_1\right)\left(\frac{\sigma}{\sigma^*} - t_2\right), \text{ and}$$

$$W^F\left(\frac{\sigma}{\sigma^*}\right) = n\left(4n+1\right)\left(n+1\right)^2\left(\frac{\sigma}{\sigma^*} - t_3\right)\left(\frac{\sigma}{\sigma^*} - t_4\right),$$

where t_1 and t_2 ($t_1 < t_2$) are the roots of the quadratic equation $W^H\left(\frac{\sigma}{\sigma^*}\right) = 0$, and t_3 and t_4 ($t_3 < t_4$) are the roots of the quadratic equation $W^F\left(\frac{\sigma}{\sigma^*}\right) = 0$. It can be checked that $t_1 t_4 = 1$ and $t_2 t_3 = 1$, and $\frac{4n+1}{4n+5} \leq t_1 < t_2 \leq 1 \leq t_3 < t_4 \leq \frac{4n+5}{4n+1}$. Now it follows that, when $\rho^2 \geq 1 - \frac{(n-2)\left(2n+1\right)^2}{n\left(4n+3\right)^2\left(n+1\right)^2}$,

$$W^H\left(\frac{\sigma}{\sigma^*}\right)\begin{cases} \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \leq t_1, \\ < 0, \text{ when } t_1 < \frac{\sigma}{\sigma^*} < t_2, \\ \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \geq t_2; \end{cases}$$

and

$$W^F\left(\frac{\sigma}{\sigma^*}\right)\begin{cases} \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \leq t_3, \\ < 0, \text{ when } t_3 < \frac{\sigma}{\sigma^*} < t_4, \\ \geq 0, \text{ when } \frac{\sigma}{\sigma^*} \geq t_4. \end{cases}$$

Note that, since $t_1 < t_2 \leq 1 \leq t_3 < t_4$, both $W^H\left(\frac{\sigma}{\sigma^*}\right)$ and $W^F\left(\frac{\sigma}{\sigma^*}\right)$ can never be negative simultaneously. Now Proposition 8 follows. ■

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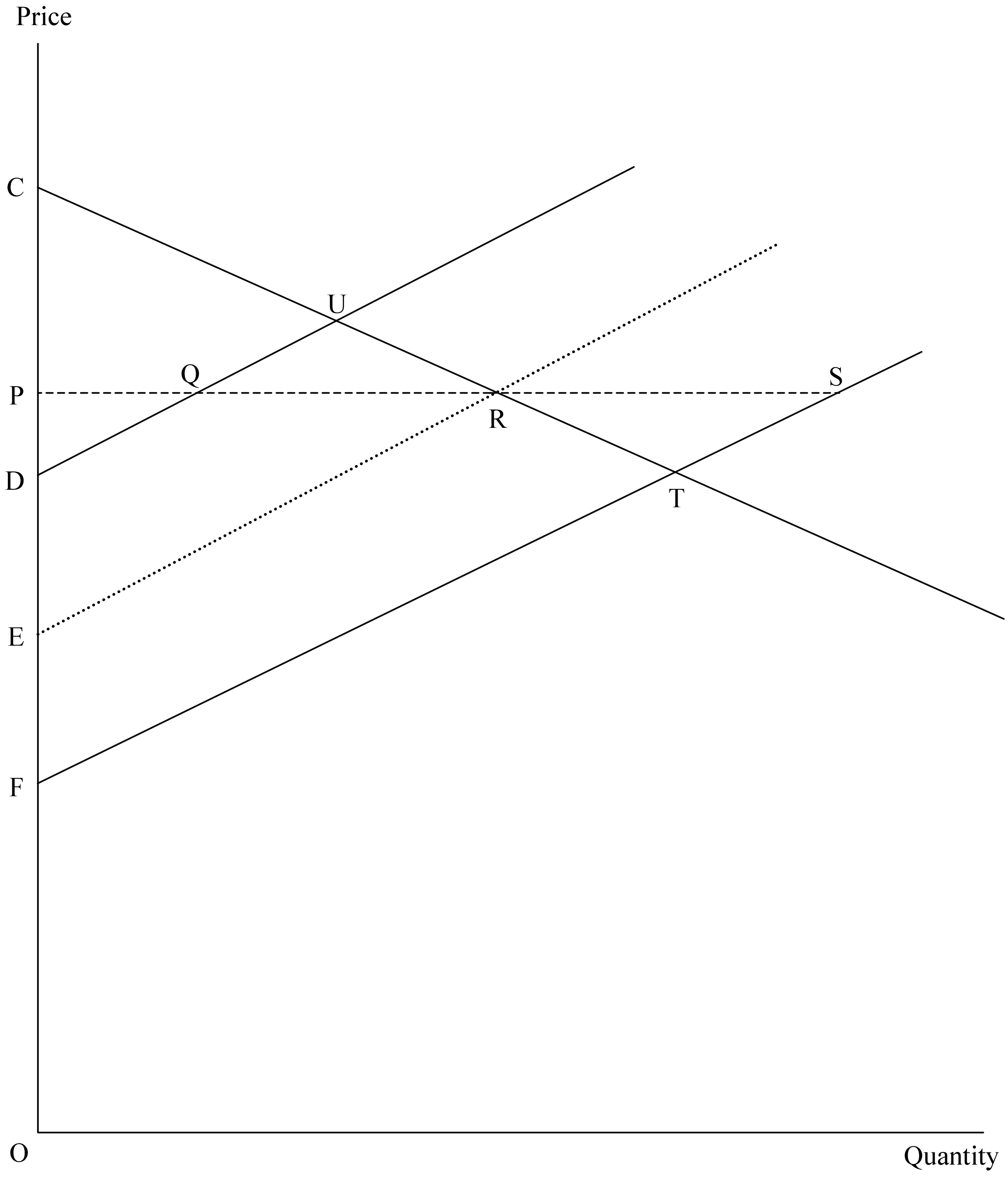

Figure 1: Social Welfare when Trade Reduces Price Variability

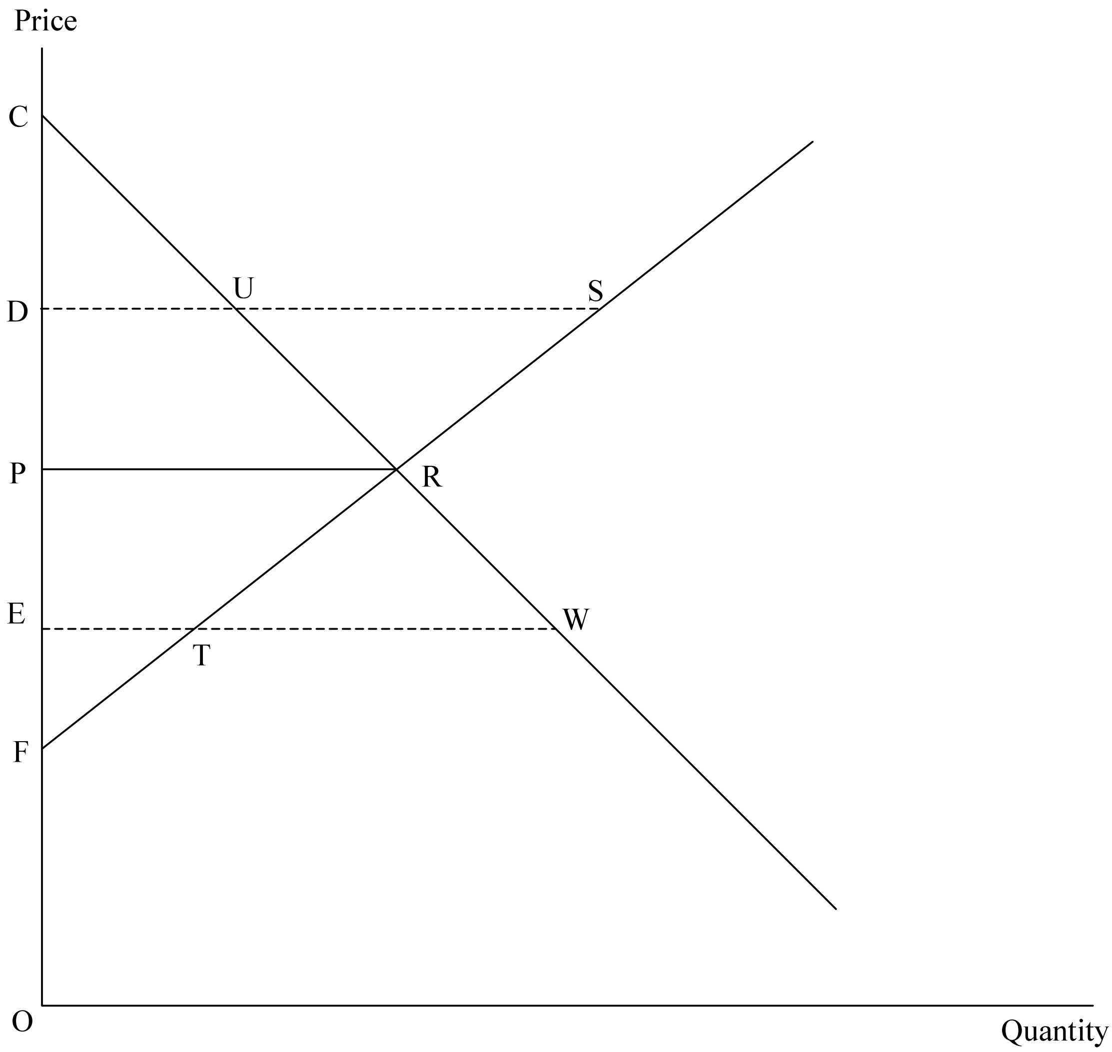

Figure 2: Social Welfare when Trade Increases Price Variability