Optimization of Regularization Parameter for GRAPPA Reconstruction

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Introduction

The effectiveness of regularization to improve SNR in parallel imaging techniques has been reported in previous works [1-2], but how to optimize the regularization parameter remains a problem. The regularization parameter controls the degree of regularization and thereby determines the compromise between SNR and artifacts. Over-regularization causes high level of artifact, while under-regularization can not prevent noise amplification. In this study, three regularization parameter choice strategies are compared in GRAPPA reconstruction: the L-curve method, the fixed singular value (SV) threshold method, and a novel discrepancy principle approach. In vivo experiment results show that the discrepancy-based parameter choice strategy significantly outperforms the others. It can automatically choose nearly optimal parameters for the reconstructions so as to achieve good compromise between SNR and artifacts.

Method

The key procedure of GRAPPA is an 'inverse/fit' process which can be simply represented by $K_{und}m = k_{ACS}$, (1)
where K_{und} is a matrix containing undersampled signals, k_{ACS} is auto-calibrating signals (ACS).
Let $K_{und} = \sum_{i=1}^{n} u_i \sigma_i v_i^T$ be the SVD of K_{und} , then the least squares solution is $m_{LS} = \sum u_i^T k_{ACS} v_i / \sigma_i$ (2)
The principle of regularization is to "smooth" the solution by truncating or damping the small SV components in Eq. (2). Various regularization strategies may be adopted, such as Truncated SVD (TSVD) and the Tiknohov regularization [3]. In this study, we discuss how to choose tailored regularization parameters for the inverse/fit in GRAPPA reconstruction.
(i). SV thresholding: simply setting a threshold of SV and eliminating all SV components below that threshold.
(ii). L-curve method [3]: plotting residual norm versus solution norm (in log scale) for different regularization parameters and locating the L-corner of the curve. This corner corresponds to the parameter chosen.
(iii). Discrepancy principle [3]: this method is based on the estimate of the errors in the right-hand side of the Eq.(1). Assuming an estimate of the errors in k_{ACS} is available, $\delta_e = \left\| k_{ACS}^{exact} - k_{ACS} \right\|$ (δ_e is the so-called "discrepancy"),
the idea of discrepancy principle is to choose the regularization parameter *r* such that $\left\| K_{und} m(r) - k_{ACS} \right\| = \delta_0 + \delta_e$, (3)
where δ_0 is the incompatibility measure of Eq. (1): $\delta_0 = \left\| K_{und} m_{LS} - k_{ACS} \right\|$.
The connection of discrepancy principle with the L-curve method can be illustrated in the plot of $\log\left\| m(r) \right\|$ versus $\log\left\| K_{und} m(r) - k_{ACS} \right\|$, as shown in Fig. 1. The L-curve choice corresponds to the corner point which has the largest curvature; while the discrepancy choice corresponds to the point where the curve intersects the line given by Eq. (3).
One practical issue when employing the discrepancy principle is how to estimate the errors in ACS lines. The noise variance ε can be calibrated by a noise scanning prior to the in vivo scanning (acquiring signals reflecting mere noise without RF transmission). δ_e can then be obtained by $\delta_e = \sqrt{n}\varepsilon$ with *n* denoting the length of k_{ACS} .
Axial and coronal brain images were acquired using an 8-element head coil array. Full dataset were acquired and later decimated off-line by various factors to simulate acceleration. GRAPPA reconstructions were performed using these artificially reduced datasets. 10 ACS lines were incorporated for harmonic fit. The relative errors in the GRAPPA-reconstructed images with respect to the corresponding full-data-reconstructed reference images are used as a yardstick of the overall image quality to compare different regularization parameter choices.

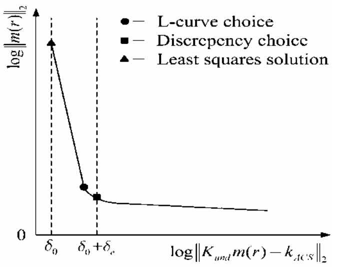

Fig. 1. Schematics of the discrepancy principle and the L-curve method.

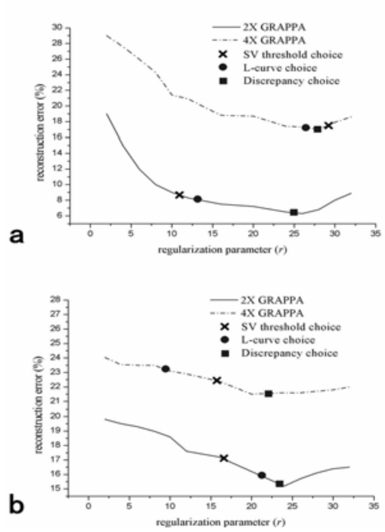

Fig. 2. Plots of reconstruction errors vs. regularization parameter for GRAPPA with TSVD regularization: (a).axial imaging; (b). coronal imaging.

Results

Fig. 2 shows the relative reconstruction errors versus regularization parameter in 2X- and 4X-GRAPPA reconstructions with TSVD regularization. For either axial or coronal case, the U-shape suggests that either over-regularization or under-regularization causes suboptimal image quality. The choices based on the SV threshold (10% of the maximal SV), the L-curve and the discrepancy principle are marked on the plots as crosses, circles and squares, respectively. Clearly, the discrepancy choices exhibit excellent adaptability and accuracy. In each reconstruction the discrepancy choice is close to the optimal choice where the relative errors are minimized. The 10% SV threshold choice is acceptable for axial imaging case, while for coronal images it leads to notable over-regularization in GRAPPA. This implies that the optimal threshold depends on specific reconstruction, and therefore a fixed threshold is not reasonable. The L-curve method tends to lead to misleading choice of parameter for every reconstruction. We further remark that L-curve is not applicable to GRAPPA. This is because usually the $\log\left\| m(r) \right\|$ - $\log\left\| K_{und} m(r) - k_{ACS} \right\|$ curve does not show a typical L-shape. The max-curvature point misleadingly suggests a fake "corner".
4X-GRAPPA coronal images are displayed in Fig. 4. Fig. 4a is the reconstruction result without regularization; Fig. 4b and Fig. 4c show the corresponding Tikhonov regularization results using the discrepancy and the L-curve, respectively. One can see that with parameters chosen with the discrepancy approach, regularization can significantly improve the SNR of the reconstruction at only a modest increase of artifacts. In Fig. 4c, the over-regularized result associated with the L-curve exhibits severe residual aliasing artifacts.

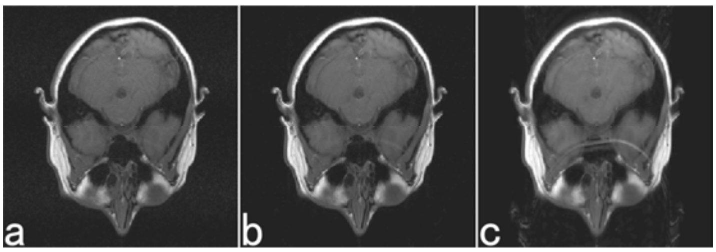

Fig. 3. 4X-GRAPPA coronal images. (a) without regularization; (b) with Tikhonov regularization + discrepancy choice; (c)with Tikhonov regularization + L-curve.

Conclusion

The discrepancy-based parameter choice strategy outperforms those based on the L-curve or on a fixed SV threshold for regularized GRAPPA reconstruction. Using discrepancy principle, adaptive regularization in GRAPPA can be realized which can automatically choose nearly optimal parameters for reconstructions.

Acknowledgement

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